MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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W. H. MAW, Esq., PRESIDENT, in the Chair.

John de Fenton, Ph.B., Seymour Avenue, Parktown, P.O. box 1050, Johannesburg, South Africa;
John Grigg, The Observatory, Thames, New Zealand; and
Robert Leetham Jones, M.A., 3 King's Bench Walk, Temple, E.C.,

were balloted for and duly elected Fellows of the Society.

Professor Julius Franz, Observatory, Breslau, Germany, was balloted for and duly elected an Associate of the Society.

The following Candidate was proposed for election as a Fellow of the Society, the name of the proposer from personal knowledge being appended :—

Alexander Davidson Fleming, Artillery Mansions, 75 Victoria Street, S.W. (proposed by the Rev. D. Fleming).

Sixty-seven presents were announced as having been received since the last meeting, including, amongst others :—

Sir W. Huggins, The Royal Society, presented by the author; Report on the Geodetic Survey of Rhodesia, presented by Sir D. Gill; 18 charts of the Astrographic Chart of the Heavens, presented by the Royal Observatory, Greenwich; and 10 charts presented by the San Fernando Observatory.

On Mr. Cowell's Discussions of Ancient Eclipses of the Sun.

By Simon Newcomb.

In his various papers discussing Greenwich observations of the Moon, Mr. Cowell has made so important a step in advance in the method of deriving results from a long series of observations, and in unravelling the intricate network of mutual relations between the quantities he has to investigate, that any conclusion he reaches in this field must carry great weight. This fact seems to render it all the more desirable to make a critical examination of his conclusions when his methods seem so open to question as do those adopted in his two papers, "On the Secular Acceleration of the Moon's Longitude and Node" and "On the Secular Acceleration of the Earth's Orbital Motion" (*Monthly Notices*, vol. lxv. p. 861 and vol. lxvi. p. 3).

I begin with a glance at the available material. Historical research has brought to light some forty or more records or statements, extending between B.C. 1069 to A.D. 200, which may be considered with a greater or less degree of probability to refer to eclipses of the Sun. In most cases the conclusion that an eclipse is referred to is undoubted; the other cases range through every degree of doubt from plausibility to absolute uncertainty. The date is frequently so doubtful that the identification of an eclipse is more or less uncertain, even when granted that an eclipse was referred to.

Several recent investigators, Airy, Hansen, Oppolzer, Ginzel, and now Cowell, have investigated various selections, generally from three to five, of these eclipses with a view of deriving corrections to the secular motions of the Moon's elements. But there is no agreement between any two of the authors except, perhaps, Airy and Hansen; and no one of the results agrees with conclusions derived from modern observations by the aid of gravitational theory. Yet there is no reason to impugn the correctness of the narrative when we measure correctness by the standard of the historian. As I have said, in the majority of cases the eclipse is so far identified as to show that the statement of the historian was well founded. But it does not follow from this that the eclipse can be utilised for any astronomical purpose. As I have repeatedly pointed out, there being no observations of times or phases, the only fact we can take as the base of a conclusion is that a well-identified eclipse was total at a known place. By a curious fatality there is always some weak point in each of the small number of cases in which this condition is presumably satisfied by the narrative. In other cases the (as it seems to me) gratuitous assumption has to be made that the eclipse was total at the point where the record was discovered, or where the historian was supposed to have lived.

In the *Monthly Notices*, vol. lxv. p. 61, Mr. Cowell discussed five of these thirty or forty eclipses, and found, on the assump-

tion that each was total at a given place, that the whole five could be represented by a correction to the secular acceleration of the Moon's node, the reality of which would involve the conclusion that, in ancient times, the annual motion of the node was about 1″·8 different from the result of gravitational theory. As the correctness of gravitational theory at the present time is proved by Brown's researches, this conclusion would imply that a law of nature was different in ancient times from what it is now.

In the *Monthly Notices* for November 1905 Mr. Cowell accepts the inadmissibility of this conclusion, and shows that his eclipses can also be represented by supposing a sidereal secular acceleration in the mean longitude of the Sun of +4″·1. In view of the fact that no such acceleration is shown by 150 years of modern observations of the Sun, and that if it exist it must be due to some other cause than gravitation, it would seem to require a good deal of proving. To test the sufficiency of Mr. Cowell's proof we must consider his evidence in detail.

His first eclipse is one recorded as occurring at Babylon −1062 July 31. Although the record does not distinctly state that the phenomenon was actually observed at Babylon, the presumption that such was the case is fairly plausible. In affording even a plausible conjecture of totality at a definite place this eclipse seems to me exceptional.

In his No. 2, recorded at Nineveh, nothing is said in the record about the eclipse being total. If we accept the modern tables as correct the southern limit of totality passed about sixty miles north of Nineveh. I see nothing unlikely in the hypothesis that this eclipse, supposing it thus to have occurred as our modern tables show, should have been recorded at Nineveh, as it must have been very striking at that point.

His eclipse No. 3 is that of Archilochus in −647. Here we have nothing to base a conclusion upon except that a certain writer vaguely mentions what undoubtedly were the phenomena of a total eclipse, but without indication when or where the phenomena occurred. It seems to be doubtful even where Archilochus lived, as Ginzel says he probably lived either at Pharos or Thasos. The identification of the description with the eclipse of −647 seems to be little more than a plausible conjecture. The only conclusion which it seems to me can fairly be drawn from the record is that at some time in the seventh century B.C. there occurred a total eclipse at some point near enough to the residence of Archilochus to admit of his hearing about it.

Eclipse No. 4 is that of Thucydides −430. This was an annular eclipse by our tables; but I see no reason for inferring that the eclipse was fully annular at Athens.

The fifth and last eclipse is that of Tertullian at Utica. Of this eclipse the writer says that it was not quite total—*extincto pene lumine*. This statement of the historian is quite in accord

with the tables if the latter indicate that the total phase passed anywhere near Utica. But in order to reach a conclusion Mr. Cowell assumes that the writer meant something different from what he says, and that the eclipse was total at Utica.

By these interpretations five of the six authorities are brought into unanimity. But how is this unanimity reached? In the most unambiguous record the interpretation put upon it is only a plausible one; the totality of the second and third eclipses at the assigned places is largely hypothetical; in the fourth, which was annular, the conclusion is doubtful; and the last witness was interpreted as meaning something different from what he said—that is to say, an eclipse which he distinctly describes as not total was assumed to be total. Unanimity reached in this way does not seem to me a sufficient basis for a conclusion so far-reaching as a secular acceleration of 4″ in the mean longitude of the Sun.

I may advert to another phase of the subject—the relation of the question to the tidal retardation of the Earth. The apparent acceleration arising from this cause results in a reaction of the Earth on the Moon causing a proportional retardation of the latter. The amount of this retardation is easily computed, and the result is found to be a diminution of the apparent increment of the acceleration of about $\frac{1}{12}$ part of its entire amount. Nothing can therefore be based on this reaction.

Two phases of the subject are alluded to by Mr. Cowell in the *Monthly Notices*, vol. lxvi. pp. 5, 6. He there alludes to my non-use of the observed magnitude of the Ptolemaic eclipses of the Moon. On this it will suffice to remark that I should regard it as assigning an unwarrantable degree of precision to the observations to suppose the magnitude of the eclipses precise to $\frac{1}{10}$ part of the Moon's diameter. This uncertainty corresponds with one of 180″ in latitude, or more than half a degree in the longitude of the node. In view of the fact that the longitude of the node at the time of these eclipses can probably be fixed from modern observations within 1′ it does not seem worth while to consider observations the probable error of which is 30′.

Mr. Cowell also alludes to the case of a Ptolemaic eclipse in which, as I said at the time, if my corrections be real, the Moon must have set at Babylon before the eclipse was total. In this he seems to be unaware that it was pointed out by Oppolzer that the record did not state that the observation was made at Babylon, as I had wrongfully supposed, but left open the easy inference that it was observed on Grecian territory.

On the whole Mr. Cowell's conclusions seem to me in need of better evidence than that which he adduces; and one might also fairly inquire whether it is sound to confine the discussion to six out of twenty or thirty real or supposed eclipses.

On Ancient Eclipses. By P. H. Cowell.

Two papers have lately appeared in the *Monthly Notices* in criticism of the conclusions that I have drawn from ancient eclipses.

I have read Mr. Nevill's paper with the greatest possible interest: he takes the standpoint that the question is one to be settled by the evidence, and not by preconceived theory, and he brings forward a large number of additional eclipses with which he tests my conclusions. I intend, therefore, to perform as soon as possible my own calculations upon these eclipses. I shall supplement Mr. Nevill's results by giving the least distance between the centres of the Sun and Moon as seen from a specified place, with symbolical corrections for the unknown quantities whose values are to be determined. At the present time, before these calculations are ready, I can offer only a few general remarks.

1. I did not choose the date −647 for the eclipse of Archilochus, but found it already assigned, and I adopted it without further inquiry. I now admit the force of Mr. Nevill's reasoning, and the eclipse must therefore be relegated to the same category as the eclipse of Agathocles.

2. Sivan was the third Babylonian month. If, therefore, July 31 was the end of Sivan, the year must have begun about May 4, and the month that began on April 4 was presumably an intercalary month. The equinox fell about March 31, and the difficulty is, therefore, only one of a few days, and not of one or two months, and consists in the question, Why was April 4 not recognised as subsequent to the equinox? But in any case errors in the calendar which might be impossible a few centuries later are not necessarily inadmissible at an earlier time. I understand that Mr. Nevill's alternative of relegating the eclipse to the twelfth century does not find favour with Mr. King. Mr. Maunder is sending an interesting note on this question to the July number of *The Observatory*.

3. Thucydides was an Athenian writing principally for Athenians. I consider the probability great that he either saw the eclipse at Athens himself, or that—if the phenomenon was different at Athens from what he described—some of his friends would have told him so.

4. In the case of the first twelve eastern eclipses given by Mr. Nevill I find in *M.N.* xxiv. p. 42, the statement that the eclipses of −708 and −600 are alone described as total. For these two eclipses, as well as for those of −180, −187, and −299, subsequently given by Mr. Nevill, my system does very well, and distinctly better than any of Mr. Nevill's three systems. The eclipse of −381, as pointed out by Mr. Nevill, fits no system of corrections.

5. Mr. Nevill makes no allusion to the lunar eclipses of the Almagest, which support my conclusions.

6. I have, of course, referred to the observations of the Sun during the last 150 years. It goes without saying that I do not maintain the value 4″·1, which I find for the secular accelerations of the Sun is correct to 0″·1. A very slightly smaller secular term might easily be masked by the errors of personality that we know to exist. I have still the further loophole that there may exist an unknown term of 300 years' period and coefficient 1″ in the Sun's longitude. (If the unknown term in the Moon's longitude is to be attributed, as Professor Newcomb once supposed, to an inequality in the Earth's rotation, a term of the required period, amplitude, and phase must necessarily exist in the Sun's longitude.)

7. I should like to raise the question as to how eclipse calculations should be presented. I prefer to resolve in longitude and latitude, at right angles to the line of sight. Mr. Nevill and many others resolve along the surface of the Earth, and tell us that according to certain tables the central line passed so many degrees north or south of a certain place. This mode of presentation is complicated by the effects of foreshortening, and we cannot tell, without considerable labour, what would be seen from the place chosen for calculation. It is obvious also that it is far easier to carry through suggested modifications of the tables in my mode of presentation than in Mr. Nevill's. I should also like to ask whether it is not worth while to give some intermediate steps in the calculation. In my first paper on solar eclipses in five pages I gave for five eclipses full formulæ and a great many intermediate steps in the calculations. Anyone who wishes to repeat my calculations can do so without any further work of reference than a table of logarithms. He can see clearly what terms I have retained and what I have neglected as unimportant. At intervals, in no case exceeding half an hour, he can say, as the case may be, "So far the calculations are numerically correct" or "An error of calculation has been introduced at this point." Lastly, if he wishes to modify my secular accelerations or other parts of my formulæ, he can do so in a few minutes. On the other hand it is impossible for me to supplement Mr. Nevill's calculations without repeating his entire work from the beginning. I very much regret this, as I have found his paper most interesting, and I deplore the unnecessary delay before I can make full use of it.

Professor Newcomb has now published a second note. It must be obvious that neither of these notes contains the features which render Mr. Nevill's paper so valuable. I find his position as to the evidence extremely difficult to understand, for he begins by saying that part of the evidence is absolutely uncertain, and he ends by asking whether it is sound not to have used more of it. I picked out what in my judgment was the best evidence, and I found it accordant upon an excessively simple geometrical hypothesis.

I need not follow him in detail through all the eclipses. I

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should never have thought of basing my conclusions on any two of them. I base my conclusions on the coincidence of all of them. A proof from coincidence is like circumstantial evidence in a criminal trial. It is unnecessary when enough direct evidence is to be had, but it is often of great force.

As for the lunar eclipses, there should be no great difficulty in estimating the magnitude to the nearest tenth of a diameter, with a probable error of one-fortieth. In ten cases magnitudes are recorded. In nine cases they indicate a large correction to the distance between the node and the Sun, always of the same sign and averaging over 1000″.

In nine of the nineteen lunar eclipses no numerical estimate of magnitude is on record. They are therefore useless for my special purpose. No. 8 is one of these. In this case Professor Newcomb has accidentally interpolated the words "at Babylon" in what purports to be a translation of Greek. No such words occur in the Greek. Other less important discrepancies between Greek and English occur in Nos. 16, 17, and 18. It does not follow that I am unaware of a mistake because I have not alluded to it, more especially as in this case eclipse No. 8, and everything connected with it, is irrelevant to the main issue.

The calculations referred to at the beginning of this note are by now sufficiently far advanced for me to make the following statements :—

1. The eclipses of −1123, −1116 and −1069 cannot possibly (that is to say, by any moderate alterations of the tables) have been total at Babylon. If, therefore, "fire in the midst of heaven" implies a total eclipse at Babylon, the date is −1062.

2. The eclipse of −609 cannot have been total at Larissa ; and the eclipse of −602 was total at Larissa according to my formulæ.

3. The eclipse of −584 will do perfectly well for the eclipse of Thales. According to my formulæ, the time of maximum eclipse occurred a few minutes before sunset.

Note on Polarisation Phenomena in the Solar Corona, 1905 *August* 30. By H. F. Newall.

The present note is presented in compliance with a request from the President. It contains an attempt at a brief *résumé* of the chief results of the observations made by the expedition of which I was in charge at Guelma relating to the phenomena of polarisation of the corona of 1905 August 30, so far as they can be stated apart from the complete measurement and study, which I hope to be able to make during the summer, of the photographic records.

It has, as I gather, been felt that the results of eclipse expeditions as given in the preliminary reports of observers are in

danger of being lost amongst the paragraphs relating to the itinerary of the expeditions and to the details of instruments used. It seems impossible to omit the itinerary from such reports ; for I think no observer on his return home can fail to wish to acknowledge publicly his obligations and indebtedness to the many persons at home and abroad from whom he has received assistance and kindness in a measure that is always surprising, however often the experience is repeated. The details about instruments are undoubtedly in very many cases of great use to observers, and can scarcely be omitted without serious loss. But without doubt the scientific interest of the observations runs the risk of being obscured in the present system of preliminary reports ; and it seems desirable to find some opportunity of discussing the observations apart from details of the expedition and simply in relation to the object of the observations, and that it should be found at a date much earlier than that of the completion of the final reports. Hence the invitation which led to the preparation of the present note.

I endeavour to summarise some of the results of the Guelma expedition by arranging them in sections dealing with (i.) the atmospheric polarisation, (ii.) the polarisation of the corona, and (iii.) general considerations.

Polarisation of the Light Scattered by the Earth's Atmosphere.

(*a*) My visual observations with a Savart polariscope showed that the plane of polarisation of the light from the sky over a field of view approximately 24° in diameter, with the corona at the centre, was either very nearly vertical or very nearly horizontal. The mean of two readings, differing by 5° from each other, indicated that the plane of polarisation was inclined either at $5\frac{1}{2}°$ to the vertical, sloping downwards to the left, or else at $5\frac{1}{2}°$ to the horizontal, sloping downwards to the right. The photographs unmistakably eliminate all doubt as between these choices, and show that the atmospheric polarisation was in the nearly horizontal plane. This observation is at variance with that of M. Salet, who, observing within thirty miles of Guelma, noted that the atmospheric polarisation was vertical.

Visual observations further proved that there was no marked change during the whole of the total phase of the eclipse at Guelma.

(*b*) Photographs taken with the Savart camera prove that the polarisation of the atmosphere was horizontal or nearly so, and that the polarisation of the corona was radial.

The polarisation is shown by the existence of marked bands—alternations of great and small intensity—over the corona and over the sky.

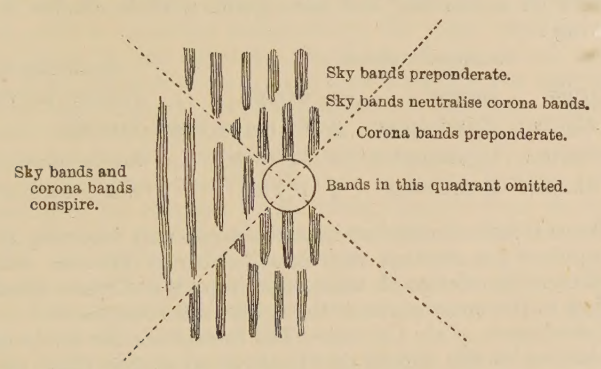
If we regard the corona as divided into four quadrants by two diameters drawn at 45° to the vertical, we may speak of the quadrants as upper, lower, right-hand, and left-hand. I refrain

at present from specifying the quadrants in terms of precise position angles to be deduced from measurements of the photographs, because I wish to avoid the need, which would almost certainly arise, of revising preliminary numbers and of giving corrections in the final discussion ; for this would lead to confusion.

The general results may be stated in this wise :—

The polarisation of the corona is shown to be approximately radial by the fact that the Savart bands (which were set very nearly parallel to the vertical diameter) in the upper and lower quadrants are out of step with the bands in the right-hand and left-hand quadrants.

The approximate horizontality of the polarisation of the atmosphere or sky is shown by the fact that the vertical bands in the outlying parts of the photograph are out of step with the coronal bands in the upper and lower quadrants, and in step with the coronal bands in the right-hand and left-hand quadrants. Diagrammatically the effects may be represented thus :—


Sky bands preponderate.
Sky bands neutralise corona bands.
Corona bands preponderate.
Sky bands and corona bands conspire.
Bands in this quadrant omitted.

Rough representation of distribution of Savart bands over sky and corona.

In the upper and lower quadrants the sky bands neutralise the corona bands at a distance of about $\frac{3}{4}°$ from the centre of the corona. This means that from the veil of illuminated sky between the observer and the corona there came as much polarised light as came from that part of the corona which is about $\frac{3}{4}°$ from the centre, in the upper and lower quadrants. This seems to me to be a result of very considerable importance in our interpretation of these and other phenomena of the corona.

(*c*) The photographs taken with the Nicol camera afford a final corroboration, if anything further is wanted, of the conclusion that the atmospheric polarisation was approximately horizontal. The actinic effect of the sky is clearly shown on the negatives by contrast with the absolutely transparent edges of the negative where the film was protected from the light by a diaphragm in the camera close to the plate. The four plates of

the series were simultaneously developed until there was danger of masking detail of the corona by bringing out the general brightness of the sky, and it should be added that in every detail of manipulation of the plates scrupulous care was given to subject them all to simultaneous and identical treatment. Now the series shows that the effect of the sky is smaller in that plate which was taken with the Nicol set to transmit the light polarised in a vertical plane than in either of the two other plates taken with the Nicol set so as to transmit light polarised in planes at 45° to the vertical on either side of it. It is thus clear that the sky polarisation was in the nearly horizontal plane.

The evidence along these lines is set forth rather fully, because it has generally, I believe, been held that the atmospheric polarisation was in a vertical plane and not of such intensity as to be likely to obscure the interpretation of the phenomena of coronal polarisation.

I have ventured in my preliminary report to "hazard numbers on memories," and have given a table similar to the following :—

		Atmospheric Polarisation.			Plane of Polarisation.
1898	India	Intense	10	Dry air	Probably not vertical.
1900	Algiers	Fairly strong	7	On the sea board	Vertical.
1901	Sumatra	Imperceptible	0	Moist air	Indeterminate.
1905	Guelma	Rather weak	3	Dry air	Nearly horizontal.

There is little doubt but that character and intensity of the atmospheric polarisation vary considerably at different eclipses, and that a knowledge of them is of importance when questions relating to the outer parts of the corona are considered.

Polarisation of the Corona.—The interest in the study of the polarisation of the corona is of course to decide what are the relative proportions of polarised and unpolarised light in the different parts of the corona.

Systematic measurement is the only way to arrive at a solution, and the photographs which we were fortunate enough to get at Guelma are well suited for this study.

All the evidence afforded by the photographs taken both with the Savart Camera and with the Nicol Camera fits in with the view that the polarised portion of the light of the corona is radially polarised.

There is strong evidence on the Savart photographs that the polarisation is quite considerable even near the limb of the Sun. On the Nicol photographs, however, it would be much more difficult to establish this point with respect to the inner corona, simply because the density of the silver deposit for the inner corona is so great that a considerable change in the intensity of the light would be needed to make much change in the density of the silver. It is a great merit of the Savart method that the

comparison of a single "polarised" photograph with an "unpolarised" photograph gives the means of estimating the difference in density in the maxima and minima in the system of bands, and so of comparing "half unpolarised + polarised" with "half unpolarised + no polarised."

In the photographs which I secured in the Sumatra eclipse of 1901 there is marked evidence of considerable polarisation close to the limb—I mean within 1′ of the limb. In photographs obtained last year in Algeria the brightness of the corona and the longer exposures given to the photographs rendered the evidence less conspicuous, but it is there, quite marked at 3′ from the limb, and I think visible nearer in.

It seems well to call attention to these points, because this evidence is, I believe, at variance with that of Mr. Perrine in Sumatra, 1901, and with that of Dr. N. E. Gilbert at Guelma, Algeria, 1905.

Polarised Spectra of the Corona.—The photograph taken with a single-prism spectrograph, which had a large double image prism in front of the camera, has two pairs of spectra; one of these pairs is polarised tangentially, the other radially. They were taken at one exposure, the image of the corona and the dark Moon being thrown on the slit, in such a way that the slit was an approximately vertical diameter of the Sun. There is a marked difference in the intensities of the tangential and radial components.* The radially polarised spectra not only extend both further from the Moon's limb and also further into the ultra-violet, but are also considerably stronger than the tangentially polarised spectra. The spectra were photographed with a narrow slit, and though comparison spectra of the Sun taken with identically the same adjustment and width of slit show the Fraunhofer lines distinctly, yet no trace of such dark lines is to be found in the spectra of the corona.

Experiments made in the spring of this year show that dark lines can be detected photographically when a mixture of one part of sky light with three parts of continuous light from a gas flame is passed into the spectrograph. Hence one is led to believe that if the polarised light in the corona were reflected sunlight it should be detected in these polarised spectra, if the

* In reference to a question raised by Professor A. Fowler at the meeting at which a short account of this paper was given, I should like to add the following note. Shortly after the eclipse, with the instrument in exactly the same state as during the eclipse observations, photographs of the sky and of the Sun were taken with the special object of showing at a glance the effect of the single prism in producing polarisation. Even had the prism been such that the angle of incidence was the angle of complete polarisation of the reflected light, then the polarised portion of the transmitted light would have been 20 per cent. of the whole transmitted. The photographs of the unpolarised sources show a just perceptible difference between the polarised components, less in fact than I should have anticipated for components in the ratio of about 2 : 3; whereas the difference in the case of the corona photographs is very marked.

proportions of polarised sunlight amounted to one-sixth of the intensity of the unpolarised light usually attributed to incandescent matter. The spectra extend to about 5′ or 7′ from the limb, viz. to a distance where the Savart photographs indicate a proportion of more like one part of polarised light to one of unpolarised (estimated), and it is difficult, therefore, not to imagine either that there is some as yet unrecognised source of polarisation which will fit in with the marked absence of Fraunhofer lines in the spectra, or else that there is some condition of scattering which somehow obliterates the dark lines from the scattered light without depolarisation. We must be careful not to adopt any explanation of the absence of dark lines that would carry with it the obliteration of such lines from direct sunlight also.

Signs of Selective Polarisation in the Corona.—The photographs got with the Nicol prism show in a preliminary inspection that different parts of the corona are very differently affected by cutting out one of the polarised components of the light. Thus it would appear that in the quadrant on the east of the south pole of the Sun there are signs that while some of the bright arches which the researches of Christie and Dyson and others seem to prove to be connected in some way with the outburst of prominences, and which may perhaps be fitly called prominential arches, are equally strong in both components of polarised light, and are thus shown to be shining with inherent light, yet other parts of the corona, notably the streamers, which become straighter as they recede from the Sun's limb, are much stronger in the radially polarised component, and are thus shown to owe their visibility probably to reflected light.

General Considerations.—It is clear that no real advance in the interpretation, or even in the statement, of the phenomena recorded in the Guelma photographs is possible until complete measurements, such as I have in part planned, are made.

If we attempt to make deductions from the equality of the polarised light scattered by the sky and by the corona at a distance of about a diameter and a half from the limb we are at once stopped by the need of quantitative knowledge. Turner's law (inverse sixth power of the distance from the centre), if it were true up to four radii, would indicate that the intensity of the corona is at that distance $\frac{1}{4096}$th part of the intensity at the limb. But the law relates to the brightness of the combined polarised and unpolarised light of the corona. For further progress we need to know the relative quantities of these components.

If we attempt to attribute the atmospheric polarisation to scattering of the integrated light of the corona, it is clear that any resultant polarisation must be due to want of symmetry in the corona. And so symmetrical a corona as that of 1905 could hardly give resultant polarisation of the relative intensity indicated by the strength of the Savart bands seen in my visual

observations. On the other hand, if the observed polarisation were attributed to secondary scattering in the atmosphere of light coming from the landscape or air in the neighbourhood of the edge of the shadow of the Moon, we should expect to have a change in the phenomena as the shadow passed over the observer. Observation showed that no such change was detected at Guelma. Any resultant polarisation would in this explanation depend also on dissymmetry in configuration of the contributing sources of illumination.

I refrain from dwelling upon some of the apparent difficulties arising in an acceptance of the idea that light pressure may be the agency by which just those particles of dust that are most active in scattering polarised light are driven out into streamers. It would appear that the particles most actively driven outwards would be those whose diameter was about one-third of the wave-length of the scattered polarised light. To adopt the view that they were driven out by the radiation pressure due to radiation of greater wave-length—let us say, ultra red—would mean that we are ready to accept the idea that the active pressure was connected with radiation known to be considerably feebler than the maximum components in the solar radiation. The particles driven out by the maximum components are already too big to scatter much polarised light, unless they simultaneously scatter unpolarised light in far larger quantities than there is evidence of in the photographic records. But it is unprofitable to try and solve what is essentially a quantitative problem by qualitative methods. I hope to revert to the subject later.

Solar Parallax Papers, No. 4. The Magnitude Equation in Right Ascension of the Etoiles de Repère. By Arthur R. Hinks, M.A.

1. The *étoiles de repère* for the reduction of the *Eros* plates were observed on the meridian at a large number of observatories, and the results were published in Paris Circulars Nos. 8 and 9.

M. Loewy published no definite catalogue of the places to be adopted as standard, preferring to leave to each observer the formation of a system for himself, at his own discretion as to the system of weights for the various observatories, and the allowance, if any, to be made for magnitude equation. As a result of this decision the published photographic places of *Eros* and the comparison stars are referred to a number of systems. The places adopted by M. Loewy for the reduction of the Paris photographs have been used also at Algiers, and in the reduction of the Catania plates at Paris. We will call this system L.

2. After a considerable part of the photographic reductions were completed, Professor Tucker published in Paris Circular No. 11 a system of places derived from a discussion of all the

meridian circle results, which he proposed for definitive adoption. He took no account of magnitude equation in the formation of this system, but published later (*Lick Observatory Bulletin*, No. 72) some reasons for thinking that the first part was free from magnitude equation, the second affected by an equation of $0^s.012$ per magnitude. We will call this system T.

In a letter to *The Observatory* (1905 July) the writer gave brief reasons for believing that the conclusion with regard to the first part was incorrect.

3. More recently Dr. Fritz Cohn has published in *Astronomische Nachrichten*, 4059–60 (1905 December) a second fundamental system, derived from the same material as Professor Tucker's, with considerable additions, but reduced to a Königsberg system of right ascensions made with the Repsold clockwork transit micrometer. Dr. Cohn claims that these observations are necessarily free from magnitude equation, and that by his reductions he has freed the other series also from its effects. We will call this system C.

4. The position at the present time is therefore this : Of the photographic observations already published Paris, Algiers, and Catania are based on system L ; Toulouse on a system differing little from it ; Bordeaux, San Fernando, and Northfield have used each an independent system.

Of the unpublished observations communicated for use in this investigation by the kindness of the Astronomer Royal, Dr. Backlund, and Professor Donner, the Greenwich results are referred to system L, the Pulkowa to system T, and the Helsingfors to an independent system.

5. Before the observations of the planet can be used for a discussion of the Solar Parallax and the mass of the Moon they all must be referred to one system. It has become now of immediate importance to inquire whether either system T (to which at present only the Pulkowa results are reduced) or system C (to which no photographic observations are as yet reduced) possesses advantages sufficient to warrant a reduction to it of all the observations ; whether it would, on the contrary, be better to reduce all outstanding series to system L, on which very much is already founded ; or whether still another fundamental system should be made from a photographic revision of one of these systems.

The present paper presents as a contribution to this inquiry the results of a determination of the magnitude equation in systems L, T, and C, derived by comparison with the deduced photographic places.

6. There are good reasons for believing that the most complete final test of freedom from magnitude equation in a series of meridian places is comparison with the results of a photographic revision of those places.*

* See papers on the magnitude equation in A. G. Zone Catalogues by the writer, *M.N.* lvii. 473 (1897 April), and by Professor Turner, *M.N.* lx. 3 (1899 November) and lxii. 3 (1901 November).

Provided that the magnitudes of the standard stars are fairly well distributed, the deduced photographic places of those stars are on the whole free from magnitude equation ; at least none is caused by the existence of magnitude equation in the adopted standard places. Hence, if we can assure ourselves that the photographic processes have introduced no magnitude equation on their own account, we may use the photographic results to determine the magnitude equation of the visual.

7. The mean photographic places of the *étoiles de repère* are found in Paris Circulars Nos. 10 and 11 (Table I. for each observatory). Each series has been compared with Paris, with the following results :—

Table I.

Paris minus Other Observatories, Photographic R.A.

	Magnitudes.							
	−6·2	6·3−6·9	7·0−7·4	7·5−7·9	8·0−8·4	8·5−8·8	8·9−9·2	9·3−.
				Paris − Algiers.				
ist I.	(5)+41	(6)+31	(15)+23	(27)+21	(53)+6	(64)−1	(43)−23	(11)−24
ist II.	(5)+51	(7)+37	(8)+11	(17)− 3	(18)−6	(24)−9	(6)−19	...
Iean	(5)+46	(13)+34	(23)+19	(44)+12	(71)+3	(88)−3	(43)−23	(11)−24
				Paris − Bordeaux.				
ist I.	(2)−17	...	(7)− 7	(12)− 2	(24)+10	(41)+ 7	(24)+ 2	(9)− 1
ist II.	(2)+22	(4)−26	(4)−10	(6)− 1	(13)+ 1	(15)− 4	(3)+11	...
Iean	(4)+ 3	(4)−26	(11)− 8	(18)− 2	(37)+ 7	(56)+ 4	(27)+ 3	(9)− 1
				Paris − Catania.				
ist I.	(3)+11	(2)−16	(9)+ 3	(21)+11	(39)+ 7	(43)+ 3	(31)+11	(8)− 4
ist II.	(4)+22	(7)+16	(5)−15	(12)+ 8	(15) 0	(16)+ 5	(3)−13	...
Iean	(7)+17	(9)+ 9	(14)− 3	(33)+10	(54)+ 5	(59)+ 3	(34)+ 9	(8)− 4
				Paris − Greenwich.				
ist I.	(3)−16	(5)− 2	(14) 0	(20)+ 3	(47)+ 2	(60) 0	(35)− 3	(8)+ 1
ist II.	(3)+11	(4)− 2	(8)−10	(12)+ 1	(16)+ 3	(16)− 5	(3)− 1	...
Iean	(6)− 3	(9)− 2	(22)− 3	(32)+ 2	(63)+ 2	(76)− 1	(38)− 3	(8)+ 1
				Paris − Helsingfors.				
ist I.	(3)+ 8	(4)+38	(9)+19	(15)+22	(22)+11	(38)+11	(20)+ 5	(10)+25
ist II.	(5)+12	(3)+ 5	(8)+13	(12)+ 4	(16)+ 8	(18)− 9	(3)−27	...
Iean	(8)+11	(7)+24	(17)+16	(27)+14	(38)+10	(56)+ 4	(23)+ 2	(10)+25
				Paris − Northfield.				
ist I.	(3)− 3	(5) 0	(12)+26	(19)+ 8	(33)+ 5	(46)+ 5	(33) 0	(7) 0
ist II.	(3)+14	(1)+ 2	(2)+ 7	(5)+ 2	(9) 0	(8)− 8	(1)−17	...
Iean	(6)+ 5	(6)+ 1	(14)+24	(24)+ 7	(42)+ 4	(54)+ 4	(34)− 1	(7) 0

	Magnitudes.							
	−6·2	6·3−6·9	7·0−7·4	7·5−7·9	8·0−8·4	8·5−8·8	8·9−9·2	9·3−.
Paris − San Fernando								
List I.	(5)+12	(6)+14	(15)+17	(27)+16	(53)+15	(66)+10	(44)− 1	(13)−11
List II.	(5)+15	(7)+ 9	(8)− 6	(17)+ 8	(19)+ 1	(24)− 3	(3)−12	...
Mean	(10)+13	(13)+11	(23)+ 9	(44)+13	(72)+11	(90)+ 7	(47)− 2	(13)−11
Paris − Toulouse.								
List I.	(5)−29	(6)+ 3	(15)+ 4	(27)− 3	(53)+10	(66)+ 3	(44)+ 3	(13)+ 4
List II.	(3)+17	(5)−25	(7)− 5	(11)+ 2	(18)− 5	(17)+ 6	(3)− 6	...
Mean	(8)−12	(11)−10	(22)+ 1	(38)− 2	(71)+ 6	(83)+ 4	(47)+ 3	(13)+ 4

The unit is $0^s.001$.

The number of separate results contributing to each mean difference is given in brackets before it.

The magnitudes are according to Tucker.

8. The comparison with Algiers shows very clearly the now well known but still unexplained systematic error, depending at any rate partly upon magnitude, which affects the whole series from that observatory. It may, perhaps, be called for convenience "objective magnitude equation." Owing to the existence of this error no use can be made of the Algiers results in this paper.

The comparison with San Fernando shows small traces of a magnitude equation which may probably be due to the fact that these plates were not reversed during measurement.

With these exceptions there is little evidence of relative magnitude equation between the different series of photographic results. And it is hard to suggest any reason why all should be affected by a similar absolute magnitude equation. We may conclude that there is in general no sensible magnitude equation in the photographic right ascensions.

9. We may, therefore, deduce the magnitude equation in system T by a similar series of comparisons. The results are given in Table II.

Table II.

System T minus Photographic R.A.

	Magnitudes.							
	−6·2.	6·3−6·9.	7·0−7·4.	7·5−7·9.	8·0−8·4.	8·5−8·8.	8·9−9·2.	9·3−.
T − Bordeaux.								
List I.	(2)−25	...	(7)−38	(12)−19	(24)− 4	(41)+ 3	(24)+12	(9)+ 7
List II.	(2)−22	(4)−41	(4)−24	(6)− 2	(13)+ 4	(15)+ 1	(3)+40	...
T − Catania.								
List I.	(5)−45	(2)−30	(13)−23	(25)−12	(44)− 2	(51)+ 3	(38)+17	(9)− 2
List II.	(4)−12	(7)+ 5	(6)−31	(12)+ 4	(15)+ 1	(16)+12	(3)+15	...

	Magnitudes. −6·2	6·3−6·9	7·0−7·4	7·5−7·9	8·0−8·4	8·5−8·8	8·9−9·2	9·3−.
T − Greenwich.								
List I.	(4) −19	(5) −18	(18) −26	(22) −18	(49) − 6	(60) + 1	(38) +11	(8) +12
List II.	(3) − 3	(2) −35	(11) −23	(12) − 3	(16) + 4	(16) + 5	(3) +29	...
T − Helsingfors.								
List I.	(4) −29	(4) +23	(12) − 4	(17) + 3	(26) + 8	(39) +11	(23) +14	(10) +34
List II.	(5) −18	(3) −15	(8) + 3	(12) + 2	(16) + 5	(19) + 3	(3) − 5	...
T − Northfield.								
List I.	(5) −12	(5) −16	(16) + 3	(25) − 2	(36) + 1	(47) + 7	(35) +11	(8) +11
List II.	(3) −12	(1) −21	(2) −14	(7) 0	(9) + 1	(8) + 1	(1) + 3	...
T − Paris.								
List I.	(5) −24	(6) −21	(15) −26	(27) −13	(53) −10	(66) 0	(44) +11	(13) +14
List II.	(5) −26	(7) −21	(8) −12	(17) − 5	(19) − 1	(24) + 4	(3) +28	...
T − San Fernando.								
List I.	(7) −21	(7) −10	(20) − 7	(35) − 1	(58) + 5	(77) + 7	(55) +10	(14) + 3
List II.	(5) −11	(7) −12	(9) −19	(17) + 3	(19) + 1	(24) + 2	(3) +16	...
T − Toulouse.								
List I.	(7) −66	(7) −21	(20) −18	(35) −17	(57) − 1	(77) + 4	(55) +13	(14) +17
List II.	(3) −20	(5) −50	(7) −16	(11) + 2	(18) − 5	(17) +13	(3) +23	...

The magnitudes are according to Tucker.

10. The results are somewhat irregular, especially for List II. But all agree in showing that system T has a considerable magnitude equation; that it is nearly the same in the two lists; and that it amounts to about $+0^{s}\cdot015$ per magnitude, the plus sign indicating that the right ascensions of faint stars are too great, or that they are observed relatively late.

11. There is some difficulty in determining the numerical value of the magnitude equation, because there are relatively few stars in the brighter groups, and accidental errors there affect the means disproportionately. Moreover it is fairly certain that the magnitude equation is not a linear function of the magnitude, even down to $9^{m}\cdot0$; but there is not sufficient material to show its true form.

All the numerical values given in this paper have been obtained by plotting the tabular results, and finding the general slope of the curve by laying a ruler on it, having regard to the numbers of stars which contribute to the different parts.

12. A similar comparison between Systems L and T shows that L has a magnitude equation in list I. about $+0^s.005$ greater than that of T, thus confirming the result noticed by Professor Turner (*Nature*, 1904 December 15), and referred to by Professor Tucker (*L.O. Bulletin*, No. 72, last paragraph). For list II. there is practically no such difference.

13. Very similar results are found for the independent systems adopted at Bordeaux, Northfield, and Helsingfors. The magnitude equation between System T and San Fernando is of the same order, but is apparently not linear.

It follows that the published photographic results are based upon adopted systems which, though differing systematically, have nearly the same magnitude equation, about $+0^s.020$ for list I. and $+0^s.015$ for list II.

14. The comparison between System C and the photographic places from each observatory was made in the same way, and the results are given in Table III.

Table III.

System C minus Photographic R.A.

	Magnitudes.							
	−6·2.	6·3−6·9.	7·0−7·4.	7·5−7·9.	8·0−8·4.	8·5−8·8.	8·9−9·2	9·3−.
				C − Bordeaux.				
List I.	(2) −62	(4) −53	(4) −46	(12) −59	(29) −52	(45) −52	(30) −55	(10) −7
List II.	(3) −48	(1) 50	(4) −54	(11) −36	(13) −31	(15) −41	(6) −31	...
				C − Catania.				
List I.	(4) −54	(10) −70	(6) −73	(24) −48	(45) −48	(65) −53	(37) −55	(9) −8
List II.	(6) −16	(4) −32	(6) −38	(16) −35	(18) −26	(21) −42	(10) −41	...
				C − Greenwich.				
List I.	(6) −36	(11) −47	(7) −48	(20) −48	(53) −53	(70) −54	(38) −58	(9) −5
List II.	(5) −24	(2) −29	(8) −40	(19) −44	(25) −36	(19) −36	(10) −40	...
				C − Helsingfors.				
List I.	(5) −23	(7) − 7	(5) −25	(17) −27	(31) −39	(38) −42	(24) −48	(11) −4
List II.	(4) −24	(5) −23	(3) −36	(15) −21	(17) −40	(14) −45	(8) −46	...
				C − Northfield.				
List I.	(6) −29	(10) −27	(8) −31	(23) −35	(42) −49	(51) −47	(29) −61	(10) −
List II.	(4) −27	(2) −38	(1) −52	(10) −29	(15) −41	(12) −44	(8) −37	...
				C − Paris.				
List I.	(5) −50	(13) −36	(8) −43	(23) −46	(59) −52	(78) −57	(46) −61	(14) −6
List II.	(8) −39	(6) −41	(7) −41	(27) −37	(30) −42	(28) −42	(14) −41	...

	Magnitudes.							
	−6·2	6·3−6·9	7·0−7·4	7·5−7·9	8·0−8·4	8·5−8·8	8·9−9·2	9·3−.
C − San Fernando.								
List I.	(9)−40	(17)−35	(11)−37	(34) 38	(65)−41	(95)−50	(57)−61	(14)−75
List II.	(8)−27	(6)−44	(9)−28	(27)−34	(30)−36	(32)−40	(13)−47	−
C − Toulouse.								
List I.	(7)−64	(16)−52	(10)−54	(33)−54	(62)−44	(92)−54	(55)−57	(14)−65
List II.	(5)−44	(5)−34	(5)−44	(22)−42	(25)−39	(25)−44	(12)−41	...

The magnitudes are according to Cohn.

Contrary to expectation, it appeared that the clockwork micrometer system for list I. showed a considerable magnitude equation, of sign opposite to that of system T, faint stars being observed early.

A result so unfavourable to the hopes which have been centred in the new method of meridian observing demanded close examination.

15. In the first place a new comparison was made with a provisional system of mean photographic right ascensions, formed by taking a simple mean of the results published by the eight observatories. The means were unweighted, since weighting would have introduced in an irregular manner the systematic differences in zero of the different series, which for our present purpose do no damage to the simple means, so long as each list is fairly completely observed at each observatory.

The results of this new comparison are, with the same magnitude groups as before :—

Table IV.

System T − Mean Photographic R.A.								
List I.	(9)−34	(16)−26	(11)−13	(34)− 8	(65)− 2	(96)+ 4	(57)+12	(14)+14
List II.	(8)−19	(6)−17	(9)−20	(28)− 4	(30) 0	(32)− 5	(15)+15	...
System C − Mean Photographic R.A.								
List I.	(9)−45	(16)−43	(11)−44	(34)−47	(65)−47	(96)−52	(57)−60	(14)−60
List II.	(8)−28	(6)−35	(9)−39	(28)−36	(30)−39	(32)−44	(15)−42	...
System C − System T.								
List I.	(9)−11	(16)−18	(11)−32	(34)−39	(65)−45	(96)−56	(57)−72	(14)−74
List II.	(8)− 9	(6)−18	(9)−19	(28)−33	(30)−38	(32)−38	(15)−57	...

The magnitude equation in system C for list I. is confirmed.

16. We must now introduce a distinction which has so far been neglected. List I. covers a range of declination from +55° to +37° ; list II. from +39° to +11°. There seems to be no

a priori reason why magnitude equation should be independent of declination—rather the reverse—but there was the fact that the general magnitude equation of system T for lists I. and II. came out almost the same ; and further, it was shown by Dr. Cohn that the differences between his final system C and his clockwork micrometer results are also independent of declination (*loc. cit.* p. 45).

But in searching for a reason why system C should have a magnitude equation for list I., and not for list II., the difference of declination suggested itself as a possible cause.

The differences which were combined to form the preceding table were therefore rearranged into three groups.

Group I. corresponds approximately to the portion of list I. north of Decl. $+49°$; these stars belong to that portion of the track of the planet pursued from 1900 October 10 to December 6.

Group II. comprises the rest of list I. between Decl. $+49°$ and $+39°$, covering the periods 1900 September 19–October 10, and December 6–29.

Group III. comprises the first part of list II. between Decl. $+38°$ and $+28°$, covering 1900 December 29 to 1901 January 25.

The mean results are as follows :—

Table V.

System T – Mean Photographic R.A.

Group I.	(3)−48	(7)−15	(5)−11	(15)− 6	(36)− 2	(52)− 1	(29)+ 8	(12)+14
Group II.	(6)−27	(9)−34	(6)−13	(19)− 9	(29)− 2	(44)+10	(28)+17	(2)+13
Group III.	(8)−19	(6)−17	(9)−20	(28)− 4	(30) 0	(32)− 5	(15)+15	...

System C – Mean Photographic R.A.

Group I.	(3)−58	(7)−29	(5)−37	(15)−37	(36)−42	(52)−52	(29)−62	(12)−61
Group II.	(6)−39	(9)−54	(6)−50	(19)−55	(29)−53	(44)−52	(28)−59	(2)−58
Group III.	(8)−28	(6)−35	(9)−39	(28)−36	(30)−39	(32)−44	(15)−42	...

A graphical determination of the magnitude equation gave

	System T. s	System C. s
Group I.	+0·010	−0·017
Group II.	+0·016	−0·002
Group III.	+0·008	−0·002

The magnitude equation in T is irregular, but not apparently related to the declination ; that of C appears strongly in the group of highest declination, scarcely at all in the others.

17. One can scarcely conclude that it really depends on the declination, since the difference of declination between groups I. and II. is not very great. But it appears certain that in group I.,

covering the period between October 10 and December 6, the system C has a magnitude equation larger than the average magnitude equation of system T, of the opposite sign. And the period in question is precisely the most important part of the whole, including opposition (October 31) and by far the greater part of the complete series east and west of the meridian.

To test this result further the differences were broken up into other groups, of which two covered between them a little more than the extent of group I.

Group A covered from October 10 to November 11, or roughly, from Decl. $+48°$ up to Decl. $+55°$.

Group B covered from November 11 to December 11; or, roughly from Decl. $+55°$ down the other side of the loop which the planet described as far as $+47°$.

System C – Mean Photographic R.A.

Group A	(1) −64	(4) −32	(3) −38	(7) −35	(21) −48	(32) −54	(21) −64	(4) −75
Group B	(3) −50	(4) −27	(2) −37	(10) −37	(17) −35	(23) −48	(11) −55	(14) −60

The magnitude equation is somewhat more strongly shown in group A than in B, which is some evidence against its intimate connection with declination.

18. As a final test the differences C minus Phot. R.A. were taken out respectively for each observatory and grouped as above. The same result came out again. With considerable variations in detail all showed more or less clearly that System C has a magnitude equation in the part covered by group I., and that it is somewhat more pronounced in the first half of that group than in the second.

19. Before accepting as final the conclusion that System C is affected in large part by a magnitude equation as great as that of System T, but of opposite sign, we must reconsider from every point of view our original proposition, that photographic places should show no magnitude equation on the mean of a long series.

Guiding error may produce photographic magnitude equation on individual plates, but we can hardly suppose it persistent with the same sign for six weeks at eight observatories.

Objective error may make the whole results of one observatory abnormal, but can hardly be supposed to exist of the same sign in eight telescopes for six weeks and then disappear.

For the same reason personality in measurement is excluded from the possibilities, even if it had not already been eliminated by reversal, except in one case.

20. There remains for consideration one qualification to our original proposition—the words, "provided that the magnitudes of the standard stars are fairly well distributed." Is it possible that there is anything so perverse in the distribution of magnitudes in the present case that the photographic methods break

down? A thorough examination of this point has been made, and the answer is decidedly in the negative.

The results of a single plate give the *slope* of the magnitude-equation curve; the zero line depends on the mean magnitude of stars on the plate. If the magnitude-equation of a system is uniform each plate should give the same slope, but each may have a different zero. A rigorous treatment of this problem would require the determination of the slope from each plate separately, and the deduction of a mean slope. In the present case this was impracticable because, unfortunately, the concluded results from each plate have not been published separately.

It is not hard to construct an arrangement of magnitudes along the path of the planet so systematically peculiar that the method followed in this paper would give a false result. The following experiment was therefore made: It was supposed that a whole series of meridian places was affected by a uniform magnitude-equation of $0^{s}\cdot017$ per magnitude, and that the stars were distributed as in the actual system. By a method which is obvious, but tedious to explain, it was found that the magnitude-equation which would be derived from the different blocks of this system would be practically identical in all, viz. that originally introduced. There is, therefore, no peculiarity in the distribution of magnitudes which can affect the result we have obtained.

21. The method withstands all the tests which have been applied to it; but it gives results which are somewhat rough, because of the accidental errors in the standard systems, apart from the systematic magnitude-equation, and because of the accidental errors in the photographic places, which may become rather large for the brighter stars. Is it possible that accidental errors have combined to produce our result?

This question has been examined by a laborious method which can be but briefly described here. The stars which would fall on a series of plates taken on centres every 2° along the path of the planet have been taken out in the usual magnitude groups for each centre and the photographic places compared with Systems T and C. The mean difference for a given magnitude group was plotted for each centre, and these points were connected by lines of a different colour for each group. For example, the group 8·0–8·4 was represented by a green line, and 8·5–8·8 by a blue. If the green line runs parallel to the blue the magnitude-equation is consistent; if the lines of different colours cross, it is not consistent.

22. First a diagram was drawn for System C—mean photographic R.A. Throughout the parts of the curves belonging to October and November the magnitude-equation was shown roughly but fairly consistently. The same thing was done for C minus Paris, Greenwich, and Toulouse, separately. The results were rougher, but the same effect could be traced.

Finally, similar curves were drawn for System T—mean photographic R.A., and the results were decidedly more consistent than for C.

23. From these investigations, which have been tested and varied in every way that can be thought of, the following results emerge :—

System T has a fairly uniform magnitude-equation of about $+0^{s}\cdot015$ per magnitude throughout list I. and the first half of list II. ; for the second half there is little material.

System L and the independent adopted systems have a magnitude-equation of about $+0^{s}\cdot020$ for list I. and $+0^{s}\cdot015$ for list II.

System C has a magnitude-equation of about $-0^{s}\cdot017$ for about half list I. and of $-0^{s}\cdot002$ for the remainder and for list II.

It seems to me, therefore, that so far as magnitude-equation is concerned System T is little better than System L, while System C is worse, because it changes so suddenly. From this point of view there would be little advantage in adopting either T or C as the standard system for the reduction of all the photographs.

My sincere acknowledgments are due to Miss Julia Bell, who has carried out very skilfully the numerical work summarised in this paper. A large portion of the expense has been borne by a grant from the Royal Society Government Grant Fund.

Cambridge Observatory:
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On some Points connected with the Determination of Orbits.

By H. C. Plummer, M.A.

1. The difference between the circular measure and the sine of an angle is an expression which occurs in several of the most important formulæ relating to elliptic motion. When the angle is large there is no difficulty in calculating this difference accurately, but when it is small ordinary logarithmic tables will not give the required accuracy without the help of some special device. Hence auxiliary tables have been published in a variety of forms, according to the purpose for which they are designed. It is possible to restrict the compass of such tables by making them merely supplementary to the ordinary logarithms by the use of some artifice. Of this nature is Tietjen's * formula, which may be expressed in the form

$$\varepsilon - \sin\varepsilon = \tfrac{4}{3}\,B\,\sin^{3}\tfrac{1}{2}\varepsilon\,(\sec\tfrac{1}{4}\varepsilon)^{2\cdot4} \quad \ldots \quad \ldots \quad (1)$$

* *A.N.* 1463 ; also Watson's *Theoretical Astronomy*, p. 343.

for which a very small table, giving log B as far as $\varepsilon = 60°$, is necessary. But the use of any special table can be avoided. Thus I have pointed out elsewhere * that the function

$$\tfrac{1}{6}\varepsilon^3 / (\sec \tfrac{1}{12}\varepsilon)^{14\cdot4} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

is a remarkably close approximation to the value of $\varepsilon - \sin\varepsilon$; indeed, over the range of ε from 0° to 70° the difference between the logarithms of the two expressions does not exceed 2 in the seventh place. A greater error may be caused by the occurrence of the power 14·4. It may be noted that if ε is less than 36° the logarithm of the required secant can be obtained from Schrön's tables (as the difference between S and T) to eight places. Within this range at least the approximation may be considered practically perfect.

2. A function of such frequent occurrence that it has been made the subject of special tables is

$$Q(\varepsilon) = \tfrac{3}{4}(\varepsilon - \sin\varepsilon)/\sin^3\tfrac{1}{2}\varepsilon.$$

This can be expressed by Tietjen's formula thus :

$$Q(\varepsilon) = B(\sec \tfrac{1}{4}\varepsilon)^{2\cdot4} \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

or by using the approximate expression (2)

$$Q(\varepsilon) = (\tfrac{1}{2}\varepsilon/\sin\tfrac{1}{2}\varepsilon)^3 (\sec \tfrac{1}{12}\varepsilon)^{-14\cdot4} \quad \ldots \quad \ldots \quad (4)$$

provided $\varepsilon < 70°$. Now the nature of the function $Q(\varepsilon)$, which is unity when $\varepsilon = 0°$, suggests that it may be represented very approximately by the form $(\sec b\varepsilon)^a$, a and b being two constants so chosen as to give the best possible result. For practical purposes, however, b must be a very simple factor. This limitation precludes a quite satisfactory approximation: no great improvement can be made on the trigonometrical factor in (3). But the case is different if we seek to represent Q not by a single term $(\sec b\varepsilon)^a$, but by a product of two such terms. This is shown by considering the logarithmic expansions of the functions involved.

3. In the first place, if μ is the modulus of common logarithms,

$$\log(\varepsilon - \sin\varepsilon) - 3\log\varepsilon + \log 6$$
$$= \mu\left[-\tfrac{1}{20}\varepsilon^2 - \tfrac{1}{1680}\varepsilon^4 + \tfrac{1}{75600}\varepsilon^6 + \tfrac{89}{3104640000}\varepsilon^8 + \ldots\right] \quad (5)$$

We have also

$$\log\sin x = \log x - \mu\Sigma\, B_{2i-1}\frac{2^{2i}}{2i \mid 2i} x^{2i} \quad \ldots \quad \ldots \quad (6)$$

$$\log\sec x = \mu\Sigma\, B_{2i-1}\frac{2^{2i}(2^{2i}-1)}{2i \mid 2i} x^{2i} \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

* *A.N* 3883

where B_1, B_3, . . . are Bernoulli's numbers, of which the first four are $\frac{1}{6}$, $\frac{1}{30}$, $\frac{1}{42}$ and $\frac{1}{30}$. By means of (5) and (6) we find

$$\log Q(\varepsilon)=\mu\left[\tfrac{3}{40}\varepsilon^2+\tfrac{11}{11200}\varepsilon^4+\tfrac{1}{56000}\varepsilon^6+\tfrac{701}{2006976000}\varepsilon^8+\ldots\right] \quad (8)$$

The object is now to combine two series of the form (7) in such a way as to reproduce the first terms of (8). Theoretically an attempt might be made to obtain the first four terms; but if simple multiples of ε only are admitted, a less perfect agreement must suffice. It is indeed remarkable enough that with the multiples $\frac{1}{4}$ and $\frac{1}{6}$ of ε three terms can be made to coincide. Thus the application of (7) gives

$$\begin{aligned}&\tfrac{24576}{7000}\log\sec\tfrac{1}{4}\varepsilon-\tfrac{17496}{7000}\log\sec\tfrac{1}{6}\varepsilon\\ &=\mu\left[\tfrac{3}{40}\varepsilon^2+\tfrac{11}{11200}\varepsilon^4+\tfrac{1}{56000}\varepsilon^6+\tfrac{17}{4838400}\varepsilon^8+\ldots\right]\\ &=\log Q(\varepsilon)-\tfrac{59}{4656960000}\mu\varepsilon^8-\ldots\end{aligned}$$

The outstanding part of the fourth term amounts to 1 in the seventh place when $\varepsilon=82°$ and the succeeding terms are still smaller. Practically throughout the first quadrant we may write

$$\begin{aligned}\log Q(\varepsilon) &= \tfrac{24576}{7000}\log\sec\tfrac{1}{4}\varepsilon-\tfrac{17496}{7000}\log\sec\tfrac{1}{6}\varepsilon\\ &=[0{\cdot}5454132]\log\sec\tfrac{1}{4}\varepsilon-[0{\cdot}3978408]\log\sec\tfrac{1}{6}\varepsilon \quad \ldots \quad (9)\end{aligned}$$

The range is greater than is actually necessary, since Q can be calculated directly when ε has a fairly large value.

4. As an example of the use of such formulæ let us consider the method of Gauss for determining an orbit when two heliocentric positions are known. This involves the solution of the equations

$$y^2=m/(l+\sin^2\tfrac{1}{2}g) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

$$y^3-y^2=m(2g-\sin 2g)/\sin^3 g=\tfrac{4}{3}mQ(2g) \quad \ldots \quad \ldots \quad (11)$$

where y is the ratio of the sector to the triangle, $2g$ is the difference of the eccentric anomalies, and l and m are given quantities. These equations can be solved by trial without using special tables such as have been given by Gauss. The natural course is to choose an approximate value of $2g$ (in the absence of more precise knowledge, $2f$, the difference of the true anomalies, may be taken) and to deduce y by means of (11). Then (10) will give an improved value of g, with which the process can be repeated.

Let $y=1/c\sinh a$. Then, by (11),

$$4mQc^3\sinh^3 a+3c\sinh a-3=0,$$

which can be compared with

$$4\sinh^3 a+3\sinh a-\sinh 3a=0,$$

giving $$mQc^2 = 1, \quad c\sinh 3\alpha = 3.$$

Now if $$e^{3\alpha} = \cot \tfrac{1}{2}\beta, \quad e^{\alpha} = \cot \tfrac{1}{2}\gamma$$
$$\sinh 3\alpha = \cot\beta, \ \sinh\alpha = \cot\gamma.$$

For (10) we may write

$$\sin^2 \tfrac{1}{2}g = l\tan^2\delta, \ y^2 = m\cos^2\delta/l.$$

Then the calculation from an assumed value of g to a better approximation is reduced to the following system of equations :—

$$\left.\begin{aligned} \cot\beta &= 3m^{\frac{1}{3}}Q^{\frac{1}{3}} \\ \tan{}^3\tfrac{1}{2}\gamma &= \tan\tfrac{1}{2}\beta \\ \cos\delta &= l^{\frac{1}{2}}Q^{\frac{1}{3}}\tan\gamma \\ \sin\tfrac{1}{2}g &= l^{\frac{1}{2}}\tan\delta \end{aligned}\right\} \quad \ldots \quad \ldots \ (12)$$

with $$y = m^{\frac{1}{2}}l^{-\frac{1}{2}}\cos\delta$$

the last step being made only after satisfactory values of g and δ are known.

5. The application of the method will be made clearer by a numerical example. The following * is chosen : $t'-t$, the interval of time, = 100 days, $\log r = 0{\cdot}221\ 6050$, $\log r' = 0{\cdot}209\ 9050$, $2f$, the angle between the two radii, = $44^\circ 25' 48''{\cdot}00$; whence

$$m = k^2(t'-t)^2/8\cos^3 f(rr')^{\frac{3}{2}} = [9{\cdot}021\ 2961]$$
$$l = (r+r')/4\cos f(rr')^{\frac{1}{2}} - \tfrac{1}{2} = [8{\cdot}603\ 5663]$$

The complete calculation by means of Bremiker's 6-figure logarithms is given below (see p. 495).

Now if, as in the given calculation, one assumed value x_1 leads to a closer approximation x_2, and x_2 similarly leads to x_3, the approximate correction to x_3 is

$$x - x_3 = (x_3-x_2)^2/\{(x_2-x_1)-(x_3-x_2)\} \quad \ldots \quad \ldots \ (13)$$

which, applied to the successive values of $\frac{1}{2}g$, gives

$$\tfrac{1}{2}g - \tfrac{1}{2}g_3 = (63''{\cdot}8)^2 \div 3658'' = +1''{\cdot}11.$$

The calculation of y can now be completed thus :—

$\frac{1}{2}g$	$12^\circ\ 5'\ 18''{\cdot}1$	δ	$46^\circ\ 16'\ 6''{\cdot}2$
L. sin $\frac{1}{2}g$	9·321018	L. cos δ	9·839654
$\frac{1}{2}$ L. l	9·301783	$\frac{1}{2}$ L. m	9·510648
		$-\frac{1}{2}$ L. l	0·698217
L. tan δ	0·019235		
		L. y	0·048519

* For the same example worked by Gauss' special tables see Bauschinger's *Tafeln zur Theoretischen Astronomie*, p. 26.

Calculation referred to on p. 494.

$2g$	44° 25′ 48″·0	48° 25′ 23″·2	β	45° 9′ 20″·0	45° 1′ 56″·2
$\frac{1}{2}g$	11 6 27·0	12 6 20·8	$\frac{1}{2}\beta$	22 34 40·0	22 30 58·1
$\frac{1}{3}g$	7 24 18·0	8 4 13·9			
			L. tan $\frac{1}{2}\beta$	9·618890	9·617571
L. sec $\frac{1}{2}g$	0·008213	0·009766	L. tan $\frac{1}{2}\gamma$	9·872963	9·872524
L. sec $\frac{1}{3}g$	0·003637	0·004323			
			$\frac{1}{2}\gamma$	36° 44′ 13″·6	36° 42′ 33″·6
LL. sec $\frac{1}{3}g$	7·560743	7·635785	γ	73 28 27·2	73 25 7·2
const.	0·397841	0·397841			
L. (1)	7·958584	8·033626	L. tan γ	0·527678	0·526136
			$\frac{1}{2}$ L. l	9·301783	9·301783
LL. sec $\frac{1}{2}g$	7·914502	7·989717	$\frac{1}{2}$ L. Q	0·009872	0·011741
const.	0·545413	0·545413			
L. (2)	8·459915	8·535130	L. cos δ	9·839333	8·839660
(2)	0·028835	0·034287	δ	46° 18′ 32″·3	46° 16′ 3″·6
(1)	0·009090	0·010805			
			L. tan δ	0·019851	0·019224
L. Q	0·019745	0·023482	$\frac{1}{2}$ L. l	9·301783	9·301783
L. 9	0·954243	0·954243			
L. m	9·021296	9·021296	L. sin $\frac{1}{2}g$	9·321634	9·321007
sum	9·995284	9·999021	$\frac{1}{2}g$	12° 6′ 20″·8	12° 5′ 17″·0
L. cot β	9·997642	9·999511			

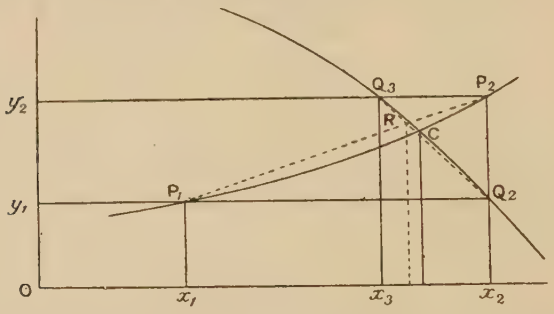
The value found by 7-figure logarithms and Gauss' tables is log $y = 0{\cdot}0485191$. The accuracy of the result is naturally greater than can be expected in general from a 6-figure computation, but the above example is merely illustrative of the method, and a more accurate calculation is therefore unnecessary.

6. It may be remarked that a still rougher computation may be of use as a preliminary step. Thus 4-figure logarithms are enough to give $\frac{1}{2}g$ in this case with an error not greater than 1′. But for the effective application of (13) it is necessary to make the two successive calculations with nearly the same order of accuracy. This formula, which is quite general, can easily be proved on the assumption that only the first powers of the errors committed are of importance. The general problem is to solve the equations

$$y = p(x),\ y = q(x),$$

where p and q are functions which involve algebraic or transcendental forms, by assuming a value x_1 and deducing x_2, which in turn is made a starting-point to obtain x_3. The two equations

may be represented graphically by the curves P_1CP_2 and Q_2CQ_3, the coordinates of whose intersection C are to be found. The method may then be translated thus: (1) with the assumed abscissa x_1 find the ordinate y_1 of P_1, (2) with this ordinate find the abscissa x_2 of Q_2 on the second curve, (3) with the abscissa x_2 find the ordinate y_2 of P_2 on the first curve, (4) with the ordinate y_2 find the abscissa x_3 of Q_3 on the second curve. It is then clear that the meaning of the correction given by (13) is to give the point R in which the straight lines P_1P_2 and Q_2Q_3 intersect. This graphical representation shows clearly the general nature of the process, and suggests in an interesting way the conditions on which its effectiveness depends. These conditions involve the slope and curvature of the lines, and are well illustrated by the special problem discussed above.



7. The equations (10) and (11), which are due to Gauss, are closely related to Lambert's theorem for elliptic motion, which is expressed by the familiar equation

$$a^{-\frac{3}{2}}k(t'-t) = (\epsilon - \sin\epsilon) - (\delta - \sin\delta) \quad \ldots \quad \ldots \quad (14)$$

where

$$\sin^2\tfrac{1}{2}\epsilon = \frac{r+r'+c}{4a}, \quad \sin^2\tfrac{1}{2}\delta = \frac{r+r'-c}{4a} \quad \ldots \quad \ldots \quad (15)$$

I have shown * that when the eccentricity is moderate and the interval between the observations is not unreasonably large, this system of equations is capable of a convenient solution and gives a practical method of determining the elements of an orbit. The usefulness of the theorem has been more generally recognised in cases of large (nearly parabolic) eccentricity. Now it is obvious that a direct calculation of $t'-t$ will not be accurate when the mean distance a is large and consequently ϵ and δ are small, especially if the chord c is also small in comparison with the radii. Hence Marth † has derived elaborate expansions in series and calculated extended tables from the results. The value of these tables is well known. Yet, though they are

* *M.N.* vol. lxiii. p. 147.

† *A.N.* 1557–1560.

doubtless convenient, they are not indispensable. It seems worthy of notice that complete accuracy can be attained by the ordinary forms of calculation.

8. With any assumed value of a, ϵ and δ can be calculated by (15); moreover, the difference of these equations gives

$$\sin \tfrac{1}{2}(\epsilon-\delta) \sin \tfrac{1}{2}(\epsilon+\delta) = c/2a \ldots \quad \ldots \quad \ldots \quad (16)$$

whence the difference $\epsilon-\delta$ can also be calculated with all necessary accuracy. Now since

$$\begin{aligned} 2 \sin \tfrac{1}{2}(\epsilon-\delta)-(\sin \epsilon-\sin \delta) &= 2 \sin \tfrac{1}{2}(\epsilon-\delta) \{1-\cos \tfrac{1}{2}(\epsilon+\delta)\} \\ &= c/a \,.\, \tan \tfrac{1}{4}(\epsilon+\delta) \end{aligned}$$

the equation (14) can now be written

$$\begin{aligned} k(t'-t) = {} & a^{\frac{1}{2}} c \tan \tfrac{1}{4}(\epsilon+\delta) \\ & + 2a^{\frac{3}{2}} \{\tfrac{1}{2}(\epsilon-\delta)-\sin \tfrac{1}{2}(\epsilon-\delta)\} \quad \ldots \quad (17) \end{aligned}$$

We now have a sum of two *positive* terms, each of which can be calculated without difficulty. By (16) this becomes

$$k(t'-t) = a^{\frac{1}{2}} c \tan \tfrac{1}{4}(\epsilon+\delta) + \tfrac{1}{24} a^{-\frac{3}{2}} c^3 \sin^{-3} \tfrac{1}{2}(\epsilon+\delta) \,.\, \mathrm{A} \ldots \quad (18)$$

where

$$\mathrm{A} = 6\{\tfrac{1}{2}(\epsilon-\delta)-\sin \tfrac{1}{2}(\epsilon-\delta)\} / \sin^3 \tfrac{1}{2}(\epsilon-\delta)$$

and tends to the value 1 when $\epsilon-\delta$ is small. It is always possible to calculate log A with the help of (2) or (9). Frequently the second term in (18) is small compared with the first term, and A is not required with great accuracy. An approximation which may be useful can be obtained by the method of § 3. This is

$$\log \mathrm{A} = 3{\cdot}6 \log \sec \tfrac{1}{4}(\epsilon-\delta) \quad \ldots \quad \ldots \quad (19)$$

which exceeds the true value by 1 in the seventh place when $\epsilon-\delta = 12°$, and will therefore in general be good enough in the case of an ellipse which is nearly parabolic.

9. It is of interest to notice the corresponding form of Euler's equation for parabolic motion. We have a infinite and $\delta = \epsilon = 0$, but at the same time the finite limits

$$a\epsilon^2 = r+r'+c, \qquad a\delta^2 = r+r'-c.$$

Since now A = 1, (17) becomes

$$k(t'-t) = \tfrac{1}{4} c \mathrm{R} + \tfrac{1}{3} c^3 / \mathrm{R}^3 \quad \ldots \quad \ldots \quad \ldots \quad (20)$$

where

$$\mathrm{R} = (r+r'+c)^{\frac{1}{2}} + (r+r'-c)^{\frac{1}{2}}.$$

This, unlike the ordinary form, is suitable for the calculation of $t'-t$ however small c may be, although not so convenient as the trigonometrical transformation employed by Encke.*

* *Berl. Jahrbuch*, 1833, p. 268

10. The numerical application of (18) is in practice very simple and convenient. For example, let $r = 1{\cdot}5$, $r' = 1{\cdot}51$, and $c = 0{\cdot}15$, and let us calculate the time-interval corresponding to these values * of a : 1·55, 10, 400 and infinity. We obtain the following results :—

a.	Period. Years.	$\frac{1}{2}\epsilon$.	$\frac{1}{2}\delta$.	$t'-t$. Days.
1·55	1·93	° ′ ″ 45 33 16·26	° ′ ″ 42 46 47·55	10·549300
10	31·6	16 19 26·55	15 30 33·49	7·865279
400	8000	2 32 49·630	2 25 23·236	7·570711
∞	∞	...	...	7·563420

In all these cases the second term on the right of (18), or (20) in the case of the parabola, could be calculated by 4-figure logarithms. Further, the value of log A was 0·0005 in the first case, and negligible for the other two elliptic orbits. The form (18) will always give accurate results, and the advantage which it offers becomes most conspicuous in those cases which have been considered to present the greatest difficulty.

University Observatory, Oxford :
1906 *June* 4.

Errors of Tabular Place of Jupiter, from Photographs taken with the Astrographic 13-inch Refractor of the Royal Observatory, Greenwich.

(*Communicated by the Astronomer-Royal.*)

In communicating the results of measures of photographs of the sixth and seventh satellites of *Jupiter* it was pointed out that in deducing the position angle and distance the error of the tabular place of *Jupiter* had been neglected.

To eliminate this error, and also that arising from any systematic error of the catalogues employed, a series of photographs of *Jupiter* was taken with the astrographic 13-inch refractor, with exposures only just long enough to give good measurable images of the known stars, *i.e.* 30^s.

In all ten photographs, taken between 1905 November 3 and 1906 February 15, have been selected and measured. Four images each of *Jupiter* and of about twelve stars were measured on each plate, the positions of the stars being derived from the *Astronomische Gesellschaft Catalogue* (Berlin zones). The deduced positions of *Jupiter* are thus affected by any systematic error of the catalogue and the error due to twenty-five years' unknown proper motion of the stars ; but, as the positions of the satellites, deduced in the same manner, are affected by the same errors, it

* For two methods of calculating $t'-t$ for the case $a = 10$ see Bauschinger's *Tafeln*, pp. 32, 33.

follows that this is the proper place of *Jupiter*, with which the places of the satellites should be compared.

The stars on the photographs were measured in the duplex astrographic micrometer, but, as the division errors of the microscope scales have not yet been determined, *Jupiter* was measured in the *Eros* micrometer. Corrections for division errors of the scale and of the réseau have been applied to the measures of *Jupiter*.

The tabular errors so obtained are given below, together with those obtained from fundamental observations made with the Transit Circle and with the Altazimuth.

Errors of Tabular Place ($T - O$)

Astrographic.				Transit Circle.			Altazimuth.		
1905.	Position of Telescope.	R.A.	Dec.	1905.	R.A.	Dec.	1905.	R.A.	Dec.
		s	″		s	″		s	″
Nov. 3	W	−·13	−0·2	Sept. 18	−·07	−0·5	Sept. 18	−·14	−1·5
21	W	−·07	+0·8	Oct. 16	−·10	0·0	Oct. 19	−·01	+0·3
23	W	−·05	+0·7	27	+·03	+0·7	Dec. 2	−·07	−0·2
29	W	−·09	+0·8	31	−·18	...	1906. Jan. 5	−·19	−0·7
1906. Jan. 19	W	−·02	−0·1	Nov. 2	−·13	−0·7	8	−·07	−0·6
19	E	−·03	−0·7	6	...	−2·2	10	−·03	−2·1
23	E	−·04	−0·6	7	+·05	−0·6	24	−·07	−0·6
29	E	+·01	−0·6	15	+·02	+1·4	Feb. 5	−·14	+0·3
Feb. 14	E	−·03	0·0	21	−·03	−0·1	9	−·02	−2·1
15	W	−·03	−0·1	23	−·06	−0·5	20	−·11	+0·5
				24	−·07	+0·4	21	−·09	−0·8
				27	−·05	+0·3			
				29	−·02	−0·9			
				Dec. 2	+·03	+0·7			
				8	−·07	−1·2			
				9	−·04	0·0			
				1906. Jan. 5	−·08	−1·3			
Mean ...		−s·048	0″·00		−s·051	−0″·29		−s·085	−0″·68

It should be noted that the photograph on November 3 is very unsatisfactory.

A correction of −″·36 for R−D has been applied to the declinations observed with the Transit Circle ; of +″·48 to those with the Altazimuth on September 18 and October 19, and of −″·13 to the remainder.

Royal Observatory, Greenwich :
1906 *June* 7.

Results of Micrometer Measures of Double Stars made with the 28-inch Refractor at the Royal Observatory, Greenwich, in the year 1905.

(*Communicated by the Astronomer Royal.*)

The measures were made with a bifilar position-micrometer on the 28-inch refractor, focal length 28 feet. The power generally employed was 670. When bright stars were observed a blue glass shade was usually employed to diminish the light and irradiation. The observations were made in variously coloured fields or in a dark field with illuminated wires. The initials in the last column are those of the observers, viz.

L.	Mr. Lewis	H.F.	Mr. Furner.
W.B.	Mr. Bowyer	Bk.	M. Van Biesbroeck.

M. Van Biesbroeck, of Brussels, spent some time at the Observatory, studying the details of various departments. On nights when the definition was not sufficiently good for measuring stars in the ordinary working list the time was spent in measuring stars from a supplementary list made up of Struve stars which have been neglected, or which require periodical observation at intervals of ten years or so, and of miscellaneous stars in which the companion is very faint. As measures of such pairs are not of immediate interest a list only of the stars observed is given here, the publication of the measures, as well as of the individual results of the observations of the other stars, being reserved for the volume of *Greenwich Observations* for 1905.

In general the present list of measures is confined to stars of which the separation is under 4″, or which show orbital movement.

Stars in the Supplementary List observed in 1905.

Struve Stars.

Σ 1955	Σ 2137	Σ 2336	Σ 2655	Σ 2734	Σ 2931
1953	2140 AB	2327	2664	2736	2932
1963	2140 AC	2364	2679	2738	2941
1964	2188	2376	2680	2761	2946
1977	2201	2381	2686	2763	2992
1992	2202	2385	2691	2831	3014
2003	2213	2390	2688	2833	3018
2005 AC	2211	2396	2692	2848	3021
2008	2217	2424	2700	2852	3028
2032 AC	2224	2426	2702	2867	3039
2040	2228	2530	2708	2877	3042
2095	2232	2585	2709	2890	3044
2104	2233 AC	2618	2713	2897	3055
2109	2233 AD	2622 AB	2714	2898	3058
2110	2264	2622 AC	2715	2902	3097
2115	2269	2631	2718	2910	3132
2113	2295	2633	2722	2916 AB	3134
2142	2311	4263	2725	2916 AC	

Otto Struve Stars.

OΣ 225, 230, 362, and 535 AC

Hough Stars.

Ho, 204, 475, 609 AC, 609 AD, 609 AE, and 614.

Miscellaneous Stars.

Burnham 151 $\frac{AB}{2}$ C

G. A. 5.

B.D. + 21°·3994 AC

Micrometric Observations of Double Stars at the Royal Observatory, Greenwich.

Star's Name.	R.A. 1900.	N.P.D. 1900.	Position Angle.	Distance.	No. of Nights	Mags.	Epoch 1905.	Obs.
	h m	° ′	°	″				
Σ 3060 ...	0 1	72 29	118·9	3·50	1	8·7 8·7	·994	W.B.
Ho. 1 ...	0 6	61 10	170·3	1·19	1	8·5 8·8	·909	L.
A.G. 56 ...	0 12	68 47	133·9	2·25	3	8·9 9·4	·639	W.B
Ho. 305 ...	0 33	65 19	193·2	5·91	2	8 11	·912	W.B.
Ho. 306 ...	0 43	64 59	165·9	1·13	2	8·5 8·8	·529	W.B.
β 495 ...	0 44	71 51	28·9	0·57	1	7·5 7·7	·887	W.B.
Σ 73 ...	0 50	66 55	25·8	0·96	2	6·2 6·8	·912	W.B.
β 302 ...	0 53	69 9	117·2	0·57	1	6·8 8·0	·887	W.B.
Ho. 310 ...	1 20	61 1	356·7	1·57	1	9·0 9·2	·830	W.B.
Ho. 9 ...	1 24	68 45	96·2	2·83	2	9 10	·912	W.B.
β 507 ...	1 30	63 44	151·3	1·86	1	8·0 11·0	·917	W.B.
Ho. 311 ...	1 46	65 48	185·7	0·49	1	7·0 7·2	·830	W.B.
Σ 194 ...	1 54	65 39	276·1	0·97	3	8·0 8·3	·056	W.B.
			272·6	1·46	1	...	·049	L.
Ho. 11 ...	1 54	56 13	141·9	4·85	1	9·0 9·4	·994	W.B.
Ho. 312 ...	2 1	64 45	345·5	0·97	2	6·5 12·0	·044	L.
Σ 226 ...	2 7	66 30	246·5	2·26	2	7·8 9·7	·039	L.
Hu. 424 ...	2 8	66 44	335·4	1·56	3	9·0 11·0	·046	L.
Ho. 216 ...	2 21	59 9	344·1	1·14	2	8·0 10·0	·030	L.
			338·3	0·95	1	...	·033	H.F.
Σ 269 ...	2 21	60 34	324·7	1·93	1	7·5 9·8	·917	W.B.
Hu. 428 ...	2 23	67 7	68·7	0·57	1	9·2 9·5	·011	L.
β 306 ...	2 38	64 48	16·7	3·01	3	6·7 11·0	·027	L.
Hu. 430 ...	2 38	69 26	188·2	0·77	1	8·5 12·8	·066	L.
Ait. 826 ...	2 39	58 56	165·7	4·16	1	8·7 11·8	·066	L.
Σ 305 ...	2 42	71 2	311·8	2·99	3	7·3 8·2	·038	W.B.
Ho. 217 ...	2 43	55 54	276·2	2·87	1	8·5 10·7	·033	H.F.

Star's Name.	R.A. 1900. h m	N.P.D. 1900. ° ′	Position Angle. °	Distance. ″	No. of Nights.	Mags.		Epoch 1905.	Obs.
β 525 ...	2 53	68 47	139·4	0·50	1	7·5	7·5	·049	L.
Σ 333 ...	2 54	69 4	198·0	1·11	3	5·7	6·0	·047	W.B.
Ho. 318 ...	2 54	73 19	205·5	3·15	1	9·1	9·1	·994	W.B.
Σ 346 ...	2 58	65 8	89·0	0·39	1	6·0	6·0	·074	W.B.
β 1030 ...	3 4	68 39	143·4	0·45	2	8·4	8·4	·078	W.B.
Ho. 14 ...	3 28	62 2	24·6	2·07	1	8·2	8·7	·994	W.B.
β 533 ...	3 29	58 39	48·6	0·80	1	7·0	7·0	·066	L.
Ho. 504 ...	3 38	54 29	182·6	0·88	1	7·8	8·0	·112	W.B.
Ho. 324 ...	3 44	75 18	332·8	0·64	3	8·1	8·3	·127	W.B.
			339·3	0·72	1	...		·033	H.F.
Hu. 24 ...	3 52	78 48	267·1	0·81	1	8·5	11·3	033	H.F.
Hu. 25 ...	3 53	78 10	324·9	0·81	1	8·6	9·1	·033	H.F.
Hu. 27 ...	3 54	80 29	214·3	0·53	1	8·1	8·5	·033	H.F.
Hu. 28 ...	3 54	78 50	340·3	1·21	1	9·0	9·2	·033	H.F.
Σ 520 ...	4 12	67 27	113·8	0·61	3	8·0	8·0	·078	W.B.
Σ 535 ...	4 18	78 51	323·1	1·64	2	6·7	8·2	·077	W.B.
Ho. 15 ...	4 18	60 7	147·4	0·72	2	8·0	8·0	·167	W.B.
β 1186 ...	4 22	79 1	172·8	0·51	1	6·8	9·7	·151	H.F.
OΣ 86 ...	4 31	70 15	54·4	0·45	1	7·5	7·5	·044	W.B.
Σ 567 ...	4 31	70 43	320·9	1·86	2	8·5	9·0	·035	W.B.
Σ 572 ...	4 32	63 18	201·5	3·76	2	6·5	6·5	·035	W.B.
Ho. 333 ...	4 38	69 53	158·6	2·17	1	9·2	9·3	·994	W.B.
Ho. 17 AB	4 53	59 8	56·5	4·36	2	7·5	10·0	·049	W.B.
Ho. 222 ...	4 53	58 34	219·3	1·87	1	7·7	10·5	·049	L.
β 1238 ...	4 55	63 36	7·2	1·56	1	8·1	11·5	·022	L.
Ho. 225 ...	5 16	77 26	111·9	0·46	3	8·0	8·1	·164	W.B.
Ho. 226 ...	5 21	62 29	238·6	0·57	4	7·0	7·0	·155	W.B.
			238·4	0·81	1	...		·153	L.
Σ 749 ...	5 31	63 8	172·2	0·66	3	7·0	7·1	·034	W.B.
Hu. 38 ...	5 39	67 8	140·8	0·48	3	8·6	8·8	·148	W.B.
			141·9	0·56	1	...		·208	H.F.
B.D. +20°·1259	6 1	69 53	187·3	2·14	1	8·7	10·0	·022	L.
			192·0	1·27	1	...		·151	H.F.
Ho. 228 ...	6 1	77 31	271·2	2·47	2	8·0	11·0	·162	W.B.
B.D. +24°·1161	6 7	65 33	178·3	1·32	2	9·0	9·2	·056	W.B.
Ho. 24 ...	6 13	80 38	156·0	4·22	1	8·0	11·0	·225	H.F.
Ho. 230 ...	6 13	76 11	61·5	1·94	2	8·2	10·5	·162	W.B

Star's Name.	R.A. 1900.	N.P.D. 1900.	Position Angle.	Distance.	No. of Nights.	Mags.	Epoch 1905	Obs.
	h m	° ′	°	″				
Σ 888 AC ...	6 14	61 31	258·2	3·16	1	7·5 9·2	·030	W.B.
B.D. +22°·1280	6 14	67 51	51·4	1·69	3	8·7 8·7	·090	W.B.
Ho. 232 ...	6 16	75 16	353·4	2·75	1	9·5 11·0	·225	H.F.
Ho. 25 ...	6 16	64 43	244·5	0·76	3	8·8 9·0	·201	W.B.
			248·9	0·90	1	...	·208	H.F.
Σ 899 ...	6 17	72 22	22·6	2·27	1	7·0 8·0	·047	W.B.
Ho. 233 ...	6 17	73 26	37·9	2·08	3	8·2 11·0	·180	W.B.
B.D. +24°·1270	6 21	65 24	208·1	2·79	2	9·0 9·1	·056	W.B.
β 1021 ...	6 25	61 32	93·6	0·75	1	8·0 9·0	·208	H.F.
OΣ 149 ...	6 30	62 38	267·8	0·80	2	6·5 9·0	·188	H.F.
B.D. +23°·1480	6 39	66 27	79·3	1·75	2	8·5 9·0	·056	W.B.
Ho. 328 ...	6 41	71 41	189·4	0·39	3	8·5 8·5	·180	W.B.
β 899 ...	6 55	71 9	265·0	0·78	1	8·7 9·3	·208	H.F.
B.D. +24°·1508	6 57	65 24	21·2	1·63	2	9·0 9·2	·056	W.B.
OΣ 165 ...	7 3	73 55	36·0	4·30	1	5·0 10·7	·167	L.
Ho. 518 ...	7 4	59 30	147·5	2·77	1	8·0 10·0	·044	W.B.
			151·6	2·82	1	...	·225	H.F.
Σ 1037 ...	7 7	62 36	295·3	0·70	2	7·1 7·1	·211	W.B.
W.B. (2) VII. 118	7 7	74 40	156·1	2·02	3	8·2 8·7	·138	W.B.
			159·0	1·87	1	...	·156	L.
OΣ 170 ...	7 12	80 32	107·7	1·41	3	7·5 7·5	·133	W.B.
			107·4	1·62	2	...	·162	L.
Σ 1066 ...	7 14	67 50	207·3	6·72	1	3·2 8·2	·183	W.B.
Ho. 243 ...	7 16	60 33	166·9	2·20	2	9·3 9·5	·230	W.B.
			166·5	2·36	1	...	·241	L.
β 1024 ...	7 16	60 31	89·4	1·46	1	9·0 11·0	·244	W.B.
B.D. +22°·1678	7 20	67 43	176·1	2·14	2	8·7 10·3	·162	W.B.
Procyon ...	7 34	84 31	5·3	4·46	1	0·5 14	·167	L.
Σ 1126 ...	7 35	84 32	143·3	1·13	1	7·2 7·5	·205	W.B.
Ho. 247 ...	7 40	68 42	108·2	0·42	4	7·5 8·0	·163	W.B.
			109·8	0·48	1	...	·208	H.F.
Ho. 36 ...	7 41	64 18	298·1	0·60	3	8·5 8·5	·145	W.B.
Σ 1196 AB	8 6	72 3	349·1	1·14	3	5·0 5·7	·234	W.B.
AC	...	...	109·6	5·51	3	5·0 6·5	·234	W.B.

Star's Name.	R.A. 1900.	N.P.D. 1900.	Position Angle.	Distance.	No. of Nights.	Mags.	Epoch 1905.	Obs.
	h m	° ′	°	″				
Σ 1196 BC	...	...	121·1	6·13	3	5·7 6·5	·234	W.B.
$\frac{AB}{2}$ C	...	...	114·8	5·45	2	5·0 6·5	·190	W.B.
B.D. +·23°·1978	8 31	66 24	184·1	1·54	3	9·2 10·2	·150	W.B.
Ho. 354 ...	8 36	63 35	181·4	0·57	1	8·2 8·8	·159	W.B.
			182·0	0·87	2	...	·255	H.F.
Ho. 251 ...	8 40	64 19	154·0	3·57	1	8·5 12·2	·277	Bk.
			153·0	4·00	1	...	·277	W.B.
Σ 1273 $\frac{AB}{2}$ C	8 41	83 13	231·6	3·33	1	3·8 7·7	·285	H.F.
B.D. + 23°·2004	8 45	66 30	258 6	2·24	1	9·0 9·1	·225	H.F.
			257·5	2·20	2	...	·269	W.B.
			257·0	2·00	2	...	·269	Bk.
Ho. 43 ...	9 13	68 47	302·7	0·47	1	8·0 8·5	·277	W.B.
			296·5	0·43	1	...	·277	Bk.
OΣ 201 ...	9 18	61 39	223·6	1·66	2	7·5 9·0	·174	W.B.
Σ 1348 ...	9 19	83 13	324·8	1·78	1	7·5 7·6	·219	L.
			319·2	1·98	2	...	·255	W.B.
Ho. 368 ...	9 32	64 12	105·0	1·01	3	8·5 8·9	·229	W.B.
			112·9	1·13	2	...	·255	H.F.
			110·5	1·09	1	...	·285	Bk.
Σ 1389 ...	9 47	62 32	307·3	2·30	2	8·0 9·0	·186	W.B.
W.B.(2)X. 128–9	10 9	71 38	6·5	1·25	3	8·0 8·5	·235	W.B.
			10·2	1·77	1	...	·277	Bk.
OΣ 215 ...	10 11	71 46	206·1	0·90	4	7·0 7·2	·238	W.B.
			206·3	1·22	1	...	·225	H.F.
Σ 1424 ...	10 14	69 39	113·3	3·60	2	2·0 3·5	·225	W.B.
			116·5	3·70	1	...	·260	Bk.
Σ 1429 ...	10 20	64 53	250·2	1·02	2	8·3 8·3	·214	W.B.
OΣ 227 ...	10 36	78 44	341·0	0·64	1	7·5 8·5	·353	L.
Σ 1523 ...	11 13	57 54	137·2	2·62	1	4·0 5·0	·189	W.B.
Σ 1527 ...	11 14	75 11	16·1	3·38	1	6·9 8·1	·260	W.B.
			16·5	3·45	1	...	·260	Bk.
Σ 1536 ...	11 19	78 55	55·0	2·36	2	3·9 7·1	·284	L.
			50·3	2·18	2	...	·255	W.B.
Lalande 21846	11 24	58 59	4·7	0·95	2	7·0 11·5	·286	L.
OΣ 235 ...	11 27	28 22	178·7	0·35	1	6 7	·277	W.B.

Star's Name.	R.A. 1900. h m	N.P.D. 1900. ° ′	Position Angle. °	Distance. ″	No. of Nights.	Mags.	Epoch 1905.	Obs.
OΣ 235 ...	11 27	28 22	182·2	0·35	1	6 7	·277	Bk.
β 603 ...	11 44	75 10	310·8	...	1	6·4 10·2	·219	L.
			319·5	1·01	1	...	·353	L.
Ho. 255 ...	12 2	68 57	133·2	2·15	1	8·2 12·3	·353	L.
Ho. 536 ...	12 16	54 26	97·4	3·84	1	8·5 9·7	·241	L.
			95·6	3·41	1	...	·320	W.B.
Σ 1639 ...	12 20	63 52	353·3	0·36	1	6·7 7·9	·372	W.B.
Σ 1643 ...	12 22	62 25	37·7	2·22	1	8·7 9·2	·260	W.B.
			38·4	2·14	1	...	·260	Bk.
Σ 1687 ...	12 48	68 13	84·1	1·23	1	5·2 8·0	·348	L.
Σ 1728 ...	13 5	71 57	193·7	0·60	1	6·0 6·0	·348	L.
			193·2	0·49	1	...	·372	W.B.
Ho 260 ...	13 19	60 15	312·9	0·76	1	8·3 8·5	·342	W.B.
β 113 ...	13 24	78 0	203·7	1·33	1	8·2 11·0	·285	H.F.
			200·6	1·75	1	...	·285	Bk.
β 801 ...	13 41	78 40	325·2	2·88	1	8·2 10·2	·405	L.
			326·6	2·83	1	...	·285	H.F.
			327·7	2·59	1	...	·285	Bk.
Σ 1785 ...	13 45	62 31	297·7	1·55	3	7·2 7·5	·313	W.B.
			304·3	1·42	1	...	·285	H.F.
			300·8	1·51	2	...	·281	Bk.
Σ 1863 ...	14 35	38 0	86·2	0·65	2	7·1 7·4	·299	W.B.
Σ1865 ...	14 36	75 51	143·8	0·37	1	3·5 3·9	·413	W.B.
Hu. 575 ...	14 38	70 5	152·5	0·69	1	9·0 9·5	·405	L.
Σ 1871 ...	14 38	39 11	294·6	1·90	2	7·0 7·0	·299	W.B.
Hu. 576 ...	14 41	69 25	190·5	5·00	1	8·2 13·0	·405	L.
Σ 1888 ...	14 47	70 29	174·1	2·41	2	4·7 6·6	·463	W.B.
β 31... ...	14 48	70 51	198·6	1·51	1	8·4 9·7	·512	W.B.
OΣ 288 ...	14 49	73 53	187·7	1·72	1	6·4 7·1	413	W.B.
Σ 1909 ...	15 1	41 57	241·9	4·32	1	5·2 6·1	·350	W.B.
			245·4	4·36	1	...	·353	L.
Σ 3090 ...	15 4	90 35	99·9	1·56	1	8·8 8·9	·449	L.
Ho. 60 ...	15 10	54 44	42·7	0·45	1	8·0 8·0	·353	L.
Σ 1932 ...	15 14	62 48	152·9	0·48	1	5·6 6·1	·350	W.B.
Σ 1938 ...	15 21	52 18	65·2	1·06	3	6·7 7·3	·356	L.
Hu. 649 ...	15 21	40 7	50·2	4·44	2	8·2 13·0	·371	L.
Σ 1941 ...	15 22	63 2	223·2	1·73	1	8·7 8·7	·367	L.
Ait. 82 ...	15 23	65 44	311·1	0·98	1	8·5 9·3	·512	W.B.

Star's Name.	R.A. 1900. h m	N.P.D. 1900. ° ′	Position Angle. °	Distance. ″	No. of Nights.	Mags.	Epoch 1905.	Obs.
OΣ 296 ...	15 23	45 39	304·6	1″48	1	...	·350	W.B.
			306·9	1·72	1	7·0 8·6	·367	L.
Σ 1950 ...	15 26	64 9	91·0	4·07	1	6·7 8·2	·367	L.
Hu. 651 ...	15 26	39 13	343·0	1·03	2	8·2 12·8	·371	L.
Σ 1956 ...	15 30	47 51	40·0	1·97	2	8·0 9·4	·310	W.B.
Σ 1959 ...	15 31	54 57	242·1	2·27	1	8·7 10·2	·277	W.B.
OΣ 298 ...	15 33	49 52	187·6	1·17	1	7·0 7·3	·350	W.B
Hu. 652 ...	15 33	40 51	175·7	0·79	1	8·5 8·8	·367	L.
Σ 1967 ...	15 39	63 23	118·0	0·72	2	4·0 7·0	·360	L.
Σ 1969 ...	15 39	29 40	49·5	0·52	1	8·0 8·7	·386	W.B.
			49·8	0·60	1	...	·386	H.F.
Hu. 657 ...	15 43	39 1	310·5	0·57	1	8·5 8·5	·367	L.
β 621 ...	15 47	45 8	54·7	0·40	1	7·5 8·0	·367	L.
Hu. 658 ...	15 51	38 16	341·9	2·63	1	8·4 13·0	·367	L.
Σ 3101 ...	15 54	92 47	68·7	2·37	1	8·2 8·5	·449	L.
Σ 1991 ...	15 54	48 3	198·0	3·56	2	8·2 9·5	·309	W.B.
Σ 2000 ...	15 56	75 45	225·9	2·46	1	8·2 9·0	·389	W.B.
OΣ 303 ...	15 56	76 27	142·5	0·80	1	7·4 7·9	·512	W.B.
Σ 2004 ...	15 59	60 52	276·1	1·45	1	8·7 9·7	·350	W.B.
Σ 2005 ...	16 0	96 1	143·6	1·43	1	6·5 8·8	·449	L.
Σ 2011 ...	16 4	60 44	70·3	2·63	1	7·2 9·8	·397	W.B.
Σ 2025 ...	16 8	42 11	165·2	2·69	1	7·6 10·9	·367	L.
Σ 2032 ...	16 11	55 54	216·3	4·80	1	5·0 6·1	·397	W.B.
Σ 2037 ...	16 14	72 22	242·9	1·45	1	9·0 9·0	·389	W.B.
Σ 2055 ...	16 26	87 48	60·5	1·18	2	4·0 6·1	·466	L.
			59·6	1·52	1	...	·468	W.B.
Σ 2061 ...	16 29	58 55	22·8	2·38	1	7·1 9·9	·389	W.B.
Σ 2080 ...	16 35	51 28	25·3	3·17	1	8·0 11·8	·389	W.B.
Σ 2084 ...	16 38	58 13	187·1	1·40	2	3·0 6·5	·527	W.B.
Σ 2091 ...	16 39	48 37	304·8	0·98	2	7·5 8·0	·385	W.B.
De. 15 ...	16 41	46 20	303·4	0·46	1	8·2 8·6	·397	W.B.
Σ 2106 ...	16 46	80 25	296·2	0·35	1	6·7 8·4	·468	W.B.
			305·2	0·32	1	...	·482	L.
Σ 2107 ...	16 48	61 10	354·0	0·34	2	6·5 8·0	·477	W.B.
			357·0	0·35	1	...	·482	L.
Σ 3106 ...	16 50	85 1	252·3	2·28	1	8·6 8·6	·545	W.B.
Σ 2112 ...	16 54	58 3	258·8	1·98	2	8·5 9·5	·393	W.B.
Σ 2114 ...	16 57	81 24	160·8	0·97	1	6·2 7·4	·389	W.B.

Star's Name.	R.A. 1900.	N.P.D. 1900.	Position Angle.	Distance.	No. of Nights.	Mags.	Epoch 1905.	Obs.
	h m	° ′	°	″				
Σ 2156 ...	17 19	90 45	34·8	3·38	2	8·3 9·0	·537	W.B.
Ho. 415 ...	17 19	64 9	331·8	1·17	2	8·0 8·7	·527	W.B.
Σ 2162 ...	17 20	53 25	279·2	1·37	1	8·7 9·2	·446	L.
B.D. +36°·2862	17 19	53 7	255·7	1·54	1	8·7 10·0	·446	L.
Σ 2168 ...	17 23	54 10	200·0	2·56	1	7·5 8·2	·397	W.B.
			197·4	2·46	1	...	·449	L.
Σ 2170 ...	17 24	79 26	270·5	3·88	1	8·5 9·0	·449	L.
Σ 2187 ...	17 31	85 48	177·0	2·99	1	8·3 9·3	·454	H.F.
Σ 2186 ...	17 31	88 56	79·6	2·77	1	7·5 7·5	·454	H.F
Σ 2200 ...	17 39	84 6	167·7	1·42	1	8·0 8·8	·454	H.F.
Σ 2205 ...	17 40	72 14	308·7	1·98	1	8·3 8·7	·512	W.B.
Σ 2206 ...	17 40	71 58	246·5	1·30	2	8·1 9·7	·477	W.B.
Σ 2215 ...	17 41	72 15	294·3	0·71	1	5·9 7·9	·512	W.B.
Σ 2212 ...	17 41	84 16	337·7	3·13	1	8·5 8·8	·482	L.
			340·3	3·13	1	...	·528	W.B.
Σ 2222 ...	17 43	75 7	59·7	2·35	2	7·5 9·2	·471	W.B.
Σ 2233 ...	17 47	87 4	73·2	2·72	1	7·5 10·3	·490	L.
Σ 2239 ...	17 48	61 43	318·6	2·55	2	8·5 9·0	·471	W.B.
Aitken 234	17 49	64 23	19·5	0·48	1	8·8 9·1	·481	L.
Σ 2240 ...	17 49	84 44	199·4	2·97	2	9·0 9·7	·479	W.B.
Aitken 235	17 49	64 59	69·7	0·46	1	7·9 8·1	·481	L.
Σ 2244 ...	17 52	89 55	278·9	1·05	1	6·9 7·1	·490	L.
Σ 2245 ...	17 52	71 39	113·3	2·70	2	7·0 7·0	·471	W.B.
Σ 2250 ...	17 54	96 51	166·6	7·45	1	8·0 9·0	·515	H.F.
Σ 2252 ...	17 54	87 57	23·6	4·01	2	8·0 8·3	·479	W.B.
Σ 2254 ...	17 54	77 33	265·1	3·43	2	8·3 8·7	·479	W.B.
Ho. 565 ...	17 59	63 56	67·1	0·55	1	8·3 8·3	·481	L.
Σ 2272 ...	18 1	87 27	178·2	2·10	12	4·5 6·0	·562	W.B.
Σ 2281 ...	18 5	86 1	Star round		1	5·7 7·2	·616	L.
			16·2	0·18	2	...	·700	W.B.
Σ 2292 ...	18 8	62 23	271·2	1·05	1	8·0 8·1	·624	L.
Σ 2294 ...	18 9	89 51	279·6	0·38	1	7·4 7·7	·616	L.
Σ 2303 ...	18 15	98 2	223·7	2·36	1	6·7 9·2	·622	W.B.
Σ 2314 ...	18 19	66 36	329·4	2·64	2	8·4 9·6	·480	W.B.
Ho. 83 ...	18 20	62 29	92·7	0·69	2	8·2 8·4	·616	L.
			87·9	0·70	1	...	·726	W.B.
Ho. 84 ...	18 20	62 37	317·7	3·81	1	9·0 11	·481	L.
			315·6	4·33	1	...	·726	W.B.

Star's Name.	R.A. 1900. h m	N.P.D. 1900. ° ′	Position Angle. °	Distance. ″	No. of Nights.	Mags.	Epoch 1905.	Obs.
Σ 2315 ...	18 21	62 40	197·3	0·27	1	7·0 8·0	·481	L.
Ho. 85 ...	18 22	61 57	194·4	5·05	2	8·0 12	·549	L.
Σ 2319 ...	18 23	70 46	189·2	5·33	2	7·2 7·7	·480	W.B.
Σ 2328 ...	18 26	60 9	72·5	3·47	2	8·0 8·3	·664	W.B.
			71·4	3·81	1	...	·481	L.
Σ 2341 ...	18 30	78 38	181·3	1·74	1	8·5 9·7	·481	L.
Σ 2347 ...	18 33	90 28	256·5	3·04	1	7·5 9·4	·622	W.B.
Ho. 437 AB	18 37	58 27	302·1	0·43	1	8·3 8·5	·674	L.
CD	...	...	336·7	3·21	1	...	·674	L.
			288·1	22·29	1	...	·674	L.
Σ 2367 AB	18 37	59 48	74·4	0·31	1	7·0 7·5	·674	L
AC	...	...	193·2	19·48	1	...	·674	L.
Σ 2397 ...	18 43	58 42	265·2	3·90	2	7·2 9·5	·664	W.B.
Σ 2402 ...	18 45	79 26	203·4	0·97	1	8·0 8·4	·701	L.
β 647 ...	18 52	76 32	9·1	0·96	2	9·0 9·2	·711	W.B
B.D. +15°·3627	18 52	74 21	33·8	1·15	1	8·5 9·5	·682	L.
Hu. 676 ...	18 53	74 18	10·8	3·15	1	7·2 10·0	·674	L.
B.D. + 14°·3718, AB	18 53	74 12	103·2	1·62	1	8·9 10·0	·682	L.
Σ 2426 BC	18 55	77 16	168·2	3·08	2	8·2 11	·692	L.
Hu. 677 ...	18 56	77 6	219·7	1·06	1	8·8 9·5	·701	L.
Ho. 93 ...	18 58	75 43	323·0	1·19	2	7·5 12·0	·711	W.B.
			317·5	1·14	1	...	·682	L.
Σ 2481 ...	19 8	51 24	220·9	4·03	1	8·0 8·0	·695	W B.
Σ 2502 ...	19 16	50 56	206·2	1·70	1	8·2 10·2	·695	W.B.
Ho. 576 ...	19 16	83 22	182·1	4·19	1	7·0 10·7	·616	L.
Σ 2505 ...	19 16	54 38	315·4	10·79	1	8·0 8·7	·695	W.B.
Σ 2525 ...	19 23	62 52	312·8	0·66	3	7·5 7·7	·761	W.B.
Σ 2599 ...	19 49	67 17	53·6	3·91	1	7·8 9·5	·539	L.
			50·7	4·14	1	...	·545	W.B.
Σ 2600 ...	19 51	67 47	54·9	3·67	1	8·3 9·7	·539	L.
			57·3	3·25	1	...	·545	W.B.
A.C. 16 ...	19 54	63 1	56·4	0·39	1	7·8 8·2	·800	W.B.
B.D. +21°·3994	19 54	68 8	277·7	1·53	1	9·0 10·4	·758	W.B.
Ho. 583 ...	19 55	68 10	255·8	1·53	1	9·0 10·7	·726	W.B
OΣ 395 ...	19 58	65 20	104·1	0·68	1	5·8 6·2	·800	W.B
Σ 2616 ...	19 58	75 42	261·8	3·85	1	6·8 9·7	·539	L.

Star's Name.	R.A. 1900.	N.P.D. 1900.	Position Angle.	Distance.	No. of Nights.	Mags.		Epoch 1905.	Obs.
	h m	° ′	°	″					
Σ 2616 ..	19 58	75 42	264·2	3·52	1	6·8	9·7	·545	W.B.
Σ 2620 ...	19 59	78 30	285·3	2·05	2	8·2	9·3	·548	W.B.
Σ 2626 ...	20 0	59 45	122·9	1·28	2	8·0	8·2	·635	W.B.
Σ 2651 ...	20 9	74 9	101·8	1·63	2	8·0	8·0	·548	W.B.
Σ 2662 ...	20 14	79 20	43·0	1·92	3	8·2	11·0	·650	W.B
Σ 2666 ...	20 15	49 35	241·6	2·48	1	6·5	8·7	·685	H.F.
Σ 2665 ...	20 15	75 57	18·8	3·65	1	6·5	9·2	·545	W.B.
Σ 2672 ...	20 17	66 34	293·7	1·01	3	8·7	8·8	·653	W.B.
Σ 2673 ...	20 18	77 0	329·9	2·39	2	8·0	9·5	·623	W.B.
Σ 2676 ...	20 19	63 11	169·2	2·35	2	7·8	10·0	·623	W.B.
β 670 ...	20 28	76 24	38·8	0·57	1	8·5	8·9	·726	W.B.
Σ 2696 ...	20 29	84 54	300·6	0·74	3	8·0	8·4	·717	H.F
Σ 2703 ...	20 32	75 37	219·1	2·37	2	7·6	7·6	·613	W.B
Σ 2701 ...	20 32	78 18	204·7	3·42	1	7·8	8·2	·813	H.F.
β 151 ...	20 33	75 45	51·3	0·25	2	4·7	6·1	·763	W.B.
Σ 2711 ...	20 35	59 51	223·8	2·14	1	8·0	9·0	·690	H.F.
			223·3	2·35	2	...		·723	W.B.
Σ 2716 ...	20 37	58 3	46·3	2·22	1	6·0	8·2	·685	H.F.
Σ 2720 ...	20 39	73 25	180·3	3·83	2	8·5	8·7	·691	W.B.
Σ 2721 ...	20 39	70 29	29·8	2·51	2	8·0	10·0	·698	W B.
Σ 2724 ...	20 40	66 26	146·7	2·50	2	8·2	8·3	·691	W.B.
Σ 2723 ...	20 40	78 3	103·1	1·35	2	7·3	8·0	·613	W.B.
Σ 2726 ...	20 42	59 39	362·9	5·31	2	4·0	9·2	·696	H.F.
Σ 2730 ...	20 46	84 0	337·2	3·50	2	7·8	7·9	·695	H.F.
Σ 2735 ...	20 51	85 51	285·3	2·07	2	6·2	7·7	·718	W.B.
			285·4	2·05	1	...		·685	H.F.
Σ 3113 ...	20 55	29 2	91·6	1·21	1	8·7	8·7	·033	H.F.
OΣ 527 ...	21 3	85 15	271·1	0·35	1	6·5	8·0	·800	W.B.
Σ 2765 ...	21 6	80 51	264·4	3·12	2	7·8	8·0	·701	W.B.
Ho. 152 ...	21 8	62 4	322·0	0·42	1	8·4	8·5	·800	W.B.
OΣ 535 ...	21 10	80 24	8·2	0·24	4	4·5	5·0	·766	L.
			7·3	0·22	8	...		·800	W.B.
			2·2	0·30	3	...		·797	H.F.
Σ 2802 ...	21 38	56 38	188·3	3·78	3	8·0	8·0	·828	W.B.
Ho. 165 ...	21 38	71 29	73·8	0·36	2	8·0	8·2	·760	W.B.
			79·8	0·39	1	...		·846	L.
Ho. 166 ...	21 40	62 38	82·2	0·33	1	7·5	7·5	·717	L.
			82·8	0·33	3	...		·735	W.B.

Star's Name.	R.A. 1900. h m	N.P.D. 1900. ° ′	Position Angle. °	Distance. ″	No. of Nights.	Mags.	Epoch 1905.	Obs.
β 89 ...	21 40	64 49	113·3	0·23	3	3·9 4·4	·797	L.
			113·3	0·26	9	...	·809	W.B.
			112·0	0·37	3	...	·800	H.F.
Σ 2824 ...	21 40	64 49	297·9	12·62	2	4·3 10·0	·739	W.B.
			297·3	12·49	1	...	·717	L.
Hu. 693 ...	21 40	39 54	226·0	1·07	1	8·7 9·2	·895	W.B.
Σ 2825 ...	21 42	89 36	117·3	0·98	1	8·0 8·2	·895	W.B.
Ho. 608 ...	21 42	63 10	123·2	0·49	2	8·2 9·7	·782	L.
Ho. 467 ...	21 46	68 13	187·2	1·27	1	8·0 10·2	·895	W.B.
Ho. 171 ...	21 48	62 41	171·8	0·81	4	8·0 8·2	·791	W.B.
Ho. 609 AB	21 51	60 45	357·5	3·16	1	9·5 9·8	·909	L.
Σ 2847 ...	21 53	93 58	301·4	1·12	1	7·6 8·0	·895	W.B.
Ho. 610 ...	21 57	63 38	224·4	0·69	1	9·0 9·2	·717	L.
Σ 2856 ...	22 1	85 37	199·4	1·27	1	8·2 8·8	·895	W.B.
Σ 2862 ...	22 2	89 55	101·7	2·58	1	7·8 7·9	·895	W.B.
Ho. 179 ...	22 8	60 20	266·1	0·45	4	8·0 8·5	·823	W.B.
Σ 2900 ...	22 19	63 39	177·8	1·38	2	...	·775	W.B.
			181·5	1·51	2	6·0 9·2	·878	L.
Espin. 147 ...	22 21	35 29	18·6	2·42	1	8·3 10·2	·895	W.B.
Σ 2905 ...	22 22	75 42	283·1	3·24	1	8·0 8·0	·909	L.
Σ 2906 ...	22 22	53 4	3·1	4·03	1	7·0 10·6	·706	H.F.
Ho. 475 ...	22 28	64 4	318·8	0·79	3	8·0 8·2	·847	W.B.
Ho. 480 ...	22 34	61 0	229·2	0·69	1	8·0 9·1	·758	W.B.
Σ 2929 ...	22 34	79 59	175·3	2·01	1	9·0 9·5	·830	W.B.
Ho. 296 ...	22 36	75 59	72·3	0·35	2	5·5 5·5	·848	W.B.
Σ 2934 ...	22 37	69 5	138·8	0·86	2	8·2 9·2	·843	W.B.
Σ 2942 ...	22 40	51 4	281·3	2·81	1	6·5 8·8	·794	H.F.
			282·6	2·67	1	...	·936	W.B.
Σ 3000 ...	23 14	65 20	50·2	3·10	1	8·7 8·8	·025	W.B.
Ho. 489 ...	23 21	62 48	238·1	0·46	3	7·5 7·5	·871	W.B.
β 720 ...	23 29	59 14	178·3	0·29	1	5·9 6·0	·887	W.B.
Σ 3026 ...	23 31	61 39	279·8	3·21	1	8·8 9·3	·025	W.B.
Ho. 201 ...	23 32	55 55	341·9	3·71	1	8·0 9·3	·936	W.B.
Ho. 203 ...	23 34	55 0	132·4	3·54	1	9·0 10·0	·936	W.B.
β 858 ...	23 36	58 0	261·1	0·55	1	7·7 8·2	·887	W.B.
β 994 ...	23 39	65 27	299·1	1·41	1	7·9 11·0	·909	L.
A.G C. 14 ...	23 39	61 12	200·7	1·24	1	5·0 9·5	·887	W.B.
Ho. 206 ...	23 54	56 17	187·8	2·06	1	8 10	·936	W.B.

The New Reduction of the Meridian Observations of Groombridge.

By Lewis Boss.

More than a year in advance of publication the Astronomer Royal at Greenwich kindly transmitted to me a manuscript catalogue of certain desired star positions newly deduced from the observations of Groombridge, 1810, by Frank W. Dyson, F.R.S., and William G. Thackeray, F.R.A.S., and later a set of revised proofs of the ledger of observations. This afforded an opportunity for comparison with my standard catalogue as well as for the extension of the same. After the work on the three higher of our five classes of stars had been substantially completed, it was found that there were available for comparison 570 well determined stars in common with positions taken from the revised catalogue of Groombridge. Accordingly a careful comparison of these stars was executed, in order to determine the relation of Groombridge's positions (revised) to those of the enlarged Standard Catalogue. The results of this comparison are outlined in the present communication.

The system of standards employed is essentially that of the *Catalogue of* 627 *Principal Standard Stars* (*Astron. Journ.*, Nos. 531–2), slightly modified in minor details since that publication. The results here presented are derived from the later of two successive approximations, each element of systematic correction having been essentially freed from the effect of other elements previous to the definitive derivation.

It became evident on examining the comparisons in R.A. that the element of difference, Stand.–Groomb., that has for its argument R.A. (Δa_a and $\Delta' a_a \tan \delta$), is, on the whole, the more important as well as complicated. Eliminating the greater part of the difference having the argument declination, and arranging in zones, we have the differences, S.–G., in Table I. In order to fit these residual differences for the treatment finally adopted they are converted into $\Delta a \cot \delta$, which in the close polar zone does not differ very greatly from $\Delta a \cos \delta$.

It is known that the transit observations of Groombridge do not usually afford the means for satisfactory determination of the polar deviation of the line of collimation. The authors call particular attention to this (Int., p. 10). Moreover the adopted values of collimation are liable to much uncertainty. We may, therefore, anticipate that there will be errors in the deduced right ascensions of the form $n \tan \delta$, or the form $n\left(1 - \tan \frac{90^\circ - \delta}{2}\right)$. The origin of the latter form of correction is to be attributed to a defective determination of the collimation, as Professor Turner has pointed out (*Monthly Notices*, vol. xlvi.), and it may exist when $n \tan \delta$ is otherwise well determined.

TABLE I.

Stand.–Groomb., $\Delta\alpha_\alpha \cot\delta$.

Zone I. 38° to 55°, decl.

R.A.	No of Stars.	$\Delta\alpha$ cot δ.	Calc.
h		s	s
23·8	26	−·009	+·007
2·1	27	+·004	+·008
4·1	16	+·027	+·007
6·0	16	−·003	+·004
8·1	11	−·021	·000
9·8	19	+·022	−·004
12·0	18	+·002	−·007
14·0	17	−·015	−·008
15·9	20	−·018	−·007
17·9	18	−·006	−·004
19·9	28	+·028	·000
21·9	24	−·002	+·004

Zone II. 56° to 66°, decl.

R.A.	No of Stars.	$\Delta\alpha$ cot δ.	Calc.
h		s	s
23·8	15	−·006	−·007
1·6	7	+·010	−·013
4·1	7	−·029	−·018
6·1	9	−·009	−·016
7·9	11	−·029	−·010
10·3	12	+·002	·000
12·5	7	+·001	+·010
13·7	9	−·004	+·013
16·3	9	+·012	+·018
18·7	9	−·016	+·014
20·1	16	+·003	+·010
22·0	15	+·002	+·001

Zone III. 67° to 77°, decl.

R.A.	No. of Stars.	$\Delta\alpha$ cot δ.	Calc.
h		s	s
0·8	9	−·026	−·021
1·8	11	−·035	−·026
3·8	2	−·057	−·031
6·0	6	−·035	−·026
8·2	7	+·033	−·014
9·9	7	−·063	−·001
12·1	7	+·009	+·016
14·3	6	+·058	+·028
16·0	6	+·002	+·031
18·4	12	+·020	+·024
19·9	11	+·030	+·016
21·8	12	−·004	+·002

Zone IV. 78° to 90°, decl.

R.A.	No of Stars.	$\Delta\alpha$ cot δ.	Calc.
h		s	s
0·4	9	+·006	−·027
2·6	3	+·053	−·040
3·9	8	−·027	−·043
6·2	6	−·048	−·035
7·5	5	−·081	−·024
10·1	6	−·008	+·003
12·2	8	+·088	+·025
13·7	4	+·082	+·036
15·8	7	+·073	+·043
18·2	5	·000	+·035
20·5	5	+·010	+·014
22·2	9	−·029	− 004

TABLE II.

Special Tabulation of $\Delta' a_a \cot \delta$, *Decl.* 56° *to* 90°.

R.A.	Zone II.	Zone III.	Zone IV.	Weighted Mean.	Calc.
h	s	s	s	s	s
0	−·024	−·040	+·003	−·023	−·027
2	−·018	−·052	+·049	−·034	−·045
4	−·061	−·076	−·031	−·054	−·050
6	−·034	−·050	−·051	−·043	−·042
8	−·042	+·027	−·083	−·029	−·023
10	+·009	−·061	−·007	−·015	+·003
12	+·022	+·021	+·091	+·045	+·027
14	+·025	+·076	+·086	+·052	+·045
16	+·044	+·021	+·077	+·046	+·050
18	+·006	+·034	+·004	+·020	+·042
20	+·015	+·038	+·011	+·022	+·023
22	−·002	−·005	−·030	−·008	−·003

Inspection of Table I. led to the conclusion that there exist sensible systematic errors in the determination of n for the transits of Groombridge, and that these errors vary with the season, or rather with the right-ascension. Accordingly it was assumed that the forty-eight mean values of $\Delta\alpha \cot \delta$ could be systematically represented by the formula

$$a \cot \delta \sin \alpha + b \cot \delta \cos \alpha + a' \sin \alpha + b' \cos \alpha,$$

in which a' and b' are coefficients of the periodic correction supposed to originate in defective determination of $n \tan \delta$, and a and b similar coefficients which would have been attributable to right-ascensions of Groombridge observed near the equator. The whole material duly weighted and treated rigorously by the method of least squares results in the following values of the respective coefficients of correction :—

$$a = +^{s}\cdot047 \pm^{s}\cdot010 \qquad a' = -^{s}\cdot042 \pm^{s}\cdot007$$
$$b = +^{s}\cdot035 \pm^{s}\cdot010 \qquad b' = -^{s}\cdot027 \pm^{s}\cdot007$$

The probable errors are large, partly because the precision of the right-ascensions is unavoidably small, and partly because of a considerable degree of indetermination between a and b compared with a' and b'. The quantities in Table I. under "Calc." are computed from the formula :—

$$+^{s}\cdot047 \cot \delta \sin \alpha + ^{s}\cdot035 \cot \delta \cos \alpha - ^{s}\cdot042 \sin \alpha - ^{s}\cdot027 \cos \alpha.$$

This is equivalent to

$$+^{s}\cdot047 \sin\alpha + ^{s}\cdot035 \cos\alpha - ^{s}\cdot042 \sin\alpha \tan\delta - ^{s}\cdot027 \cos\alpha \tan\delta$$

applied directly to the right-ascensions of the Catalogue.

The probable error of a right-ascension of Groombridge based on five observations is about $\pm^{s}\cdot04$, so that the representation of the observed values of $\Delta\alpha \cot\delta$ by the calculated values is, perhaps, as good as could have been expected. The validity of the adopted formula may be examined in another way. At 45° of N.P.D. the entire correction becomes $+^{s}\cdot005 \sin\alpha + ^{s}\cdot008 \cos\alpha$, and this seems fairly consistent with the procedure adopted by the authors of employing as additional clock-stars "all stars between 35° and 52° N.P.D. for which reliable places could be found" (Int., p. x); since it may be assumed that the standard places adopted (largely those of Newcomb's fundamental catalogue) are measurably free from error of the form Δa_{a}. A further test of the applicability of the formula can be found by subtracting the effect of a and b in zones II., III., and IV. of Table I., leaving only the observed effect of periodic terms in a' and b'. The result of this process is shown in Table II. The column under "Mean" exhibits the observed coefficients of correction in the term $\Delta' a_{a} \tan\delta$ collected into weighted means from the three zones—*i.e.* N.P.D. 34° to the pole. These should be consistent with the adopted correction,

$$-^{s}\cdot042 \sin\alpha - ^{s}\cdot027 \cos\alpha$$

computed for two-hour intervals under the caption, "Calc." The real existence of a general term of this nature appears to be clearly indicated. This implies that if, with the adopted values of clock-corrections and $n \tan\delta$, the right-ascensions of equatorial stars observed by Groombridge should be computed, they would require a correction of approximately $+^{s}\cdot047 \sin\alpha + ^{s}\cdot035 \cos\alpha$. Some support of this probability is indicated by means of the table, pp. xxxv–xxxvii, in the introduction of the Catalogue, though no solid conclusion can be drawn from numbers subject to the large and irregular discrepancies which characterise these.

The most that we can hope for in a case of this kind is to remove a general systematic error, trusting that the still outstanding errors may be treated as if they were of the same general nature as casual errors of observation. The value of systematic corrections in general seems to pertain less to their influence upon computations concerning an individual star than to those which relate to large numbers of stars in a limited region of sky. Thus a systematic error of $^{s}\cdot02 \sec\delta$ in the right-ascension of a close circumpolar star employed in deducing $n \tan\delta$ for a transit instrument could not be matter of much concern; but a systematic error of this amount applicable to all the polar stars of several contiguous hours of right-ascension, employed in

computation of $n \tan \delta$ for many nights, would be of serious consequence in accurate work.

The treatment of the right-ascensions for systematic errors depending on declination as the argument presents no points of special interest in this instance. The results are exhibited in Table III. The residuals of this table are, for the most part,

TABLE III.

Corrections Dependent on Decl., $\Delta\alpha_\delta$.

Mean δ.	No. of Stars.	Observed $\Delta\alpha_\delta \cos\delta$.	Observed $\Delta\alpha_\delta$.	Mean δ.	No. of Stars.	Observed $\Delta\alpha_\delta \cos\delta$.	Observed $\Delta\alpha_\delta$.
°		s	s	°		s	s
86	34	+·016	+·23	55	64	+·001	+·002
81	42	+·006	+·04	50	72	−·006	−·009
75	36	+·005	+·02	45	71	−·030	−·043
70	48	+·009	+·025	40	87	−·007	−·009
65	52	−·006	−·015				
60	58	−·016	−·032				

really minute. Those at 60° and 45° may indicate local effects due to slight inequalities in the form of the pivots, and some weight has been attached to this idea in drawing the curve of correction, of which the following exhibits the general result :—

δ.		δ.	
°	s	°	s
90	$+\cdot010 \sec\delta$	65	−·012
85	$+\cdot010 \sec\delta$	60	−·018
80	$+\cdot010 \sec\delta$	55	−·010
80	+·058	50	−·018
75	+·035	45	−·028
70	+·009	40	−·018

The exigencies of space will not permit a full exposition of the method of dealing with the personal equation for magnitude affecting the right-ascensions of Groombridge. I may venture to say, however, that it has been possible to construct a catalogue of standard right-ascensions that appears, both as to positions and motions, to be practically free from the effect of errors dependent on the magnitude of the stars observed. Additional evidence, as it accumulates from time to time, serves to strengthen this opinion. Therefore I have assumed that the magnitude-equation for the catalogue of Groombridge can be determined with a sufficient approximation to the truth by comparison with my standard catalogue corrected for its magnitude-equation, the right-ascensions of Groombridge having been first corrected for terms already treated in the foregoing paragraphs. A summary of results divided into zones is found in Table IV.

In spite of the fact that the precision in determination of time of transit should be decidedly smaller for stars of the declination here concerned than for equatorial stars, the effect of mean magnitude-equation is clearly indicated in each of the zones, and quite satisfactorily so in the mean. The adopted formula of correction for magnitude is

$$-^{s}\cdot 0107\ (M-3^{m}\cdot 5),$$

wherein M denotes the magnitude of an observed star in the scale of a normal uranometry in which the logarithm of the light-ratio is supposed to be 0·36, about that of the historic scale. If the light-ratio is taken as 0·40, the formula of correction becomes $-^{s}\cdot 0119\ (M-3^{m}\cdot 5)$. The column under "Calc." in the last group of Table IV. (general means) is computed from the adopted equation; but it includes also a constant, $-^{s}\cdot 004$, to eliminate the effect of constant error in the preliminary values of $\Delta\alpha_\delta$.

TABLE IV.

Magnitude-equation, $\Delta\alpha_m$.

Zone I. 38° to 47°, decl.			Zone II. 48° to 57°, decl.		
Magn.	No. of Stars.	$\Delta\alpha_m$.	Magn.	No. of Stars.	$\Delta\alpha_m$.
m		s	m		s
2·8	12	−·002	2·3	11	+·008
4·0	29	+·003	4·0	26	−·022
5·0	64	−·032	5·0	51	−·026
6·0	46	−·019	6·0	36	−·014
7	8	−·087	7	9	−·002

Zone III. 58° to 63°, decl.			Decl., 38° to 63°, Mean.			
Magn.	No. of Stars.	$\Delta\alpha_m$.	Magn.	No. of Stars.	$\Delta\alpha_m$.	Calc.
m		s	m		s	s
2·7	10	+·063	2·5	23	+·018	+·007
4·0	8	−·009	4·0	63	−·009	−·009
5·0	18	−·023	5·0	133	−·029	−·020
6·0	30	−·037	6·0	112	−·023	−·031
			7·0	17	−·042	−·041

An entirely critical computation concerning the precision of the right-ascensions of Groombridge is scarcely warranted, because of large and irregular discordances in the observations, and because of the varying number of transit threads (often only one) employed by Groombridge in his observations. However I have arrived at the following rough expression for the p.e. of the right-ascensions:

$$\text{p.e.} = \left\{\sqrt{(^{s}\cdot 018)^2+\frac{(^{s}\cdot 090)^2}{n}}\right\}\sec\delta,$$

in which n indicates the number of observations for a given star. This is really a low degree of precision, easily accounted for by the small number of transit wires usually employed, and by the unavoidable uncertainty of the constants of reduction.

The comparison of the north polar distances of the Catalogue of Groombridge with the standard offers no point of special difficulty. Table V. contains the result for the differences under the argument, declination, in the sense of declination, Stand.–Groomb. The computed probable errors of the group-values of $\Delta\delta_\delta$ fluctuate between $\pm''\cdot06$ and $\pm''\cdot04$.

TABLE V.

Observed Values of $\Delta\delta_\delta$.

Mean δ.	No. of Stars.	$\Delta\delta_\delta$.	Curve.	Mean δ.	No. of Stars.	$\Delta\delta_\delta$.	Curve.
86°	40	+″·09	+″·10	55°	64	−″·02	−′·01
80	35	+·27	+·14	50	73	−·03	·00
75	35	−·33	−·18	45	70	+·03	+·04
70	45	−·15	−·16	40	82	+·13	+·08
65	54	−·16	−·09				
60	59	+·02	−·04				

The division errors were determined by the authors from comparison with Newcomb's fundamental Catalogue of the preliminary values of N.P.D. deduced in the two positions of the circle, east and west. The employment of this method appears to have been judicious. The only material differences from the standard catalogue adopted in this paper are found in the vicinity of the pole; and this is as we might have expected. Within 10° from the pole the authors were unable to determine the division error, on account of lack of material; from 10° to 25° from the pole the error could be determined in only one position of the instrument (Int. p. xxxix).

TABLE VI.

Observed Values of $\Delta\delta_\alpha$.

Mean R.A.	No. of Stars.	$\Delta\delta_\alpha$.	Mean R.A.	No. of Stars.	$\Delta\delta_\alpha$.
h		″	h		″
0	62	−″·02	12	41	−″·06
2	50	−·18	14	43	+·06
4	32	+·20	16	45	+·13
6	40	−·02	18	48	−·01
8	35	−·04	20	64	·00
10	46	−·11	22	63	+·01

For that part of the comparison which has for its argument R.A. we have the results in Table VI. The observed values of $\Delta\delta_a$, in the sense of corrections to the declinations of Groombridge, are small, and they do not indicate a systematic difference from the standard of an appreciable amount. Following my practice in all similar instances I have deduced the following formula of correction for the declinations of Groombridge :—

$$-''\cdot 03 \sin a - ''\cdot 00 \cos a.$$

Comparing 1150 pairs of observations taken at random from the star-ledger I find the casual p.e. of a single N.P.D. of Groombridge to be about $\pm''\cdot 86$. This in connection with the 569 comparisons with the Standard Catalogue (due effect accorded to the weight of the standard for 1810) results in the following expression for the probable error of a catalogue N.P.D. :—

$$\text{p.e} = \sqrt{(\pm''\cdot 18)^2 + \frac{(''\cdot 86)^2}{n}},$$

wherein n represents the number of observations. Assuming $\pm''\cdot 30$ as the p.e. of the unit of weight, the weights in N.P.D. for Groombridge are :

Observations.	Weight.	Observations.	Weight.	Observations.	Weight.
1	·1	5	·5	9	·8
2	·2	6	·6	10	·8
3	·3	7	·6	15	1·0
4	·4	8	·7	20	1·2

Although the zenith-distances of Groombridge depend upon circle-readings, for which only two microscopes were employed, the precision of the resulting N.P.D.'s, indicated in the foregoing, ranks with the best among those of the first half of the nineteenth century—the more important series of fundamental determinations excepted. For instances, this precision is fully twice that which attaches to the original edition of Groombridge's Catalogue ; it is more than twice that of Taylor's Madras Catalogue (new reduction by Downing) ; it is very distinctly superior to that of the first Armagh Catalogue (1840) ; and, indeed, it seems to be slightly superior to Johnson's observations contained in the first Radcliffe Catalogue (1845).

In R.A. a few of the differences, Stand.–Groom., are large—even larger than could be readily attributed to the large probable error of observation. This is naturally accounted for in the fact that, in addition to the casual probable error of observation, there are important systematic errors peculiar to the observations of individual nights. The unlucky massing of these in the case of certain stars is what one might naturally expect.

The distribution of errors in N.P.D. is remarkably satis-

factory. Only one such difference calls for special remark. The uncorrected difference, Stand.–Groomb., for Groomb. 3709 is $+3''{\cdot}37$. The authors remark (footnote, p. 95) that Groombridge appears to have considered that 3707 and 3709 lie on the same parallel; for in every instance he has only one circle-reading for the two stars. All nearly contemporary evidence shows that 3709, the following and fainter star of the pair, must have been, in 1810, fully $3''$ north of Groomb. 3707. Consequently it would appear that the circle-readings in question refer exclusively to the brighter and preceding star.

A Simple Method of Obtaining an Approximate Solution of Kepler's Equation. By Arthur A. Rambaut, M.A., D.Sc., F.R.S.

In the *Monthly Notices*, vol. l. p. 301, I have described a method of solving Kepler's equation, viz.

$$m = u - e \sin u,$$

by means of a prolate trochoid.

Six years later the same method was described by Mr. Plummer in the *Monthly Notices*, vol. lvi. p. 317, but no question of priority need arise, as the principle of the method is 200 years older than either of us, being contained in the thirty-first proposition of the first book of Newton's *Principia*.

The object of the present note is to point out another way in which the solution may be very simply effected by the help of the involute of the circle.

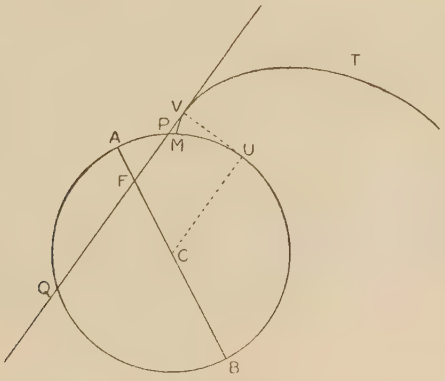


Fig. 1.

Let us take a circle, AMB, fig. 1, with centre C and any radius AC, and lay off the arc AM corresponding to the angle m. From M draw portion of the involute MVT, which can be done

very accurately by mechanical means. Divide AC at F so that FC $= e$. From F draw a tangent to the involute at V cutting AC the circle in P and Q. Then the angle AFV $= u$.

For the normal to the involute at V touches the circle at U, where the radius CU is parallel to the tangent FV. But

$$\mathrm{AM} = \mathrm{AU} - \mathrm{MU} = \mathrm{AU} - \mathrm{VU} = \mathrm{AU} - \mathrm{FC} \sin \mathrm{ACU},$$

or dividing by the radius,

$$m = \mathrm{ACU} - \frac{\mathrm{FC}}{\mathrm{AC}} \sin \mathrm{ACU}$$

Hence ACU $=$ AFV $= u$.

Now if the circle be graduated and numbered from 0° at A through P to 180° at B, and also from 0° at B through Q to 180° at A, the arcs AP and BQ may be read off directly, and we have

$$\mathrm{AU} = \tfrac{1}{2}(\mathrm{AP} + \mathrm{BQ})$$

Hence by taking the mean of the readings of the circle at the points P and Q, where it is cut by the tangent to the involute, we obtain the value of u. It may also be remarked that if u_0 is an approximate value of u thus obtained, and if m_0 be computed from the expression *

$$m_0 = u_0 - e \sin u_0$$

we have

$$\Delta u = \frac{m - m_0}{1 - e \cos u_0}$$

Δu being the correction of the first order required by u_0. But if the radius of the circle be taken as unity we have

$$\mathrm{FV} = 1 - e \cos u_0$$

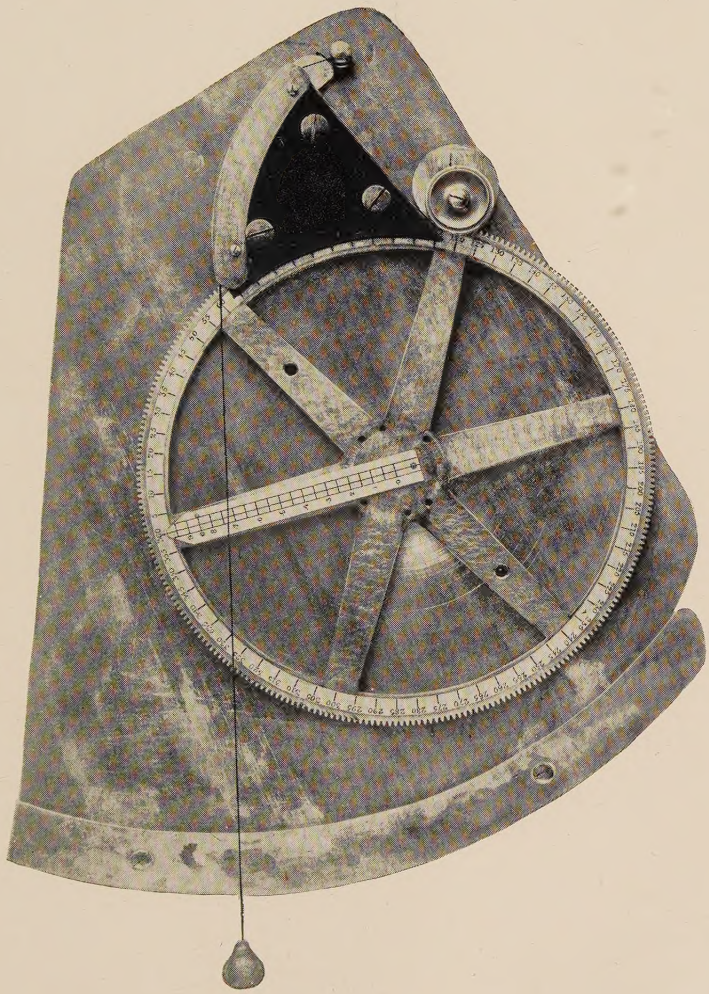
Hence

$$\Delta u = \frac{m - m_0}{\mathrm{FV}}$$

Fig. 2 (Plate 2) is a reproduction of a photograph of a very simple instrument constructed on the above principle, by which the value of u may be obtained to within one-tenth of a degree in elliptic orbits of any eccentricity.

For this instrument I have adapted a small setting circle taken from the old "Jones" Meridian Circle of the Radcliffe Observatory. This circle is 5·5 inch in diameter, and is very neatly divided to half-degrees on silver. As the graduations are not numbered, as suggested above, but run continuously through

* Compare Dr. See's paper entitled "A General Method for Facilitating the Solution of Kepler's Equation by Mechanical Means," *Monthly Notices*, vol. lv. p. 425.



MECHANICAL SOLUTION OF KEPLER'S EQUATION. A. A. RAMBAUT.
FIG. 2.

360°, it is necessary to subtract 90° from the mean of the readings at P and Q in fig. 1.

The instrument has been constructed for me by a skilful watchmaker—Mr. H. Minn, of Oxford. The involute is cut out of a plate of ebonite, and seems to be very accurate in figure. The radius is divided to hundredths on a thin silvered plate screwed on to the face of the circle. The tangent is represented by a fine string winding around the involute, passing across the face of the circle and kept taut by a small weight which hangs over the edge of the brass plate, supported on three feet, which forms the base of the instrument.

With this apparatus a first approximation to u in ellipses of any eccentricity can be found with great rapidity. The advantages it seems to possess are : (1) the involute can be drawn very accurately by means of a string, or watch-spring, coiled round a cylinder ; (2) a very small portion of the involute is required, only so much of it, in fact, as corresponds to a radian in the circle ; (3) all angles are read off on the circle, which may be divided with any degree of accuracy required.

It would be unbecoming of me if in praise of this instrument I were to indulge in the enthusiastic language which Dr. See bestows on Waterson's Curve of Sines method (see *Monthly Notices*, vol. lvi. p. 54). I hope, however, that Dr. See may be induced to give this new method a trial, and in that case I shall be disappointed if he does not come to prefer it to any other.